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The Sixth-Grade Students' Conceptualization of Angle

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Abstract

This study investigated sixth-grade students' conceptualization of angles. A structured interview was implemented with eight students to examine how they verbally described angles, represented them in figures, and interpreted them in real-life contexts such as slope, turn, and openness. The findings revealed that students primarily understood angles through static components such as corners and intersection, consistent with the standard definition of angles. Only one student demonstrated a more comprehensive understanding by relating angles to slope and space contexts through reflective abstraction of the formal angle definition. Educational implications are discussed, highlighting that most students struggled to connect angles across different contexts, such as slope and turn, despite the inclusion of real-life examples and physical experiences in the primary school curriculum. Practical implications emphasize the importance of facilitating reflective abstraction through multiple representations and contexts of angles in classroom instruction.

Keywords: mathematics education, middle school, sixth grade, angle concept, conceptualization

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Introduction

Understanding the concept of angle is essential for understanding, structuring, and constructing geometric shapes (Battista, 2007; Clements & Battista, 1992; Van Hiele, 1985). In geometric reasoning such as conceptualization of the "spatial relationships within and among shapes" (p. 843) the angle concept is a key attribute to be identified and analyzed in given shapes (Battista, 2007). Therefore, students must develop a certain level of abstraction in their understanding of angles to attain relational conceptualization and interpretation of other mathematical concepts. In addition, when students begin to view angles as attributes of geometric shapes, they move beyond the initial stage of geometric knowledge development where they interpret shapes solely based on their appearance (Clements et al., 2004; Van Hiele, 1985). Considering the importance of understanding the angle concept in relational understanding of geometric shapes and advanced mathematics subjects such as trigonometry, the concept deserves a critical place in elementary years for students to explore and develop a robust understanding (Browning et al., 2008; Clements & Burns, 2000).

The concept of angle is described in multiple ways including three common descriptions given in the educational literature (e.g., Browning et al., 2008; Keiser, 2004; Marjanovic, 2007). These are two rays with a common end point (intersection), the amount of turn (rotation), and the space between two rays meeting at a common endpoint (Browning et al., 2008; Keiser, 2000, 2004; Mitchelmore & White, 1998). These descriptions introduce the concept in multiple contexts: as a geometrical shape, a type of measure, and a dynamic mathematical construct (Tuluk, 2015). In this scope, the concept of angle can be represented in three ways in teaching: an intersection, a turn, and a wedge, respectively (Keiser, 2000).

A standard definition of angle as *"two rays with a common endpoint"* (Keiser, 2000, p. 510), demonstrates angles in the form of a geometric shape and a static form (Browning et al., 2008; Kaur, 2020; Mitchelmore & White, 2000). Another description of angles in the static form is *the space between two rays* which is represented as a wedge (Keiser, 2004; Mitchelmore & White, 2000). Understanding angles in the form of a

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wedge could help students gradually develop “the idea of degree” (p. 285) by focusing on the space (region) between the two rays as a measurable attribute through non-standard units of wedges (Browning et al., 2008). However, this description may be misinterpreted by students who might focus on the width of the area between the rays (Keiser, 2004) rather than “the amount of space that you turn between two lines” (Browning et al., 2008, p. 287). Describing angles in terms of the amount of turn, such as “the rotation around a fixed point” (Browning et al., 2008, p. 283) or “a rotation about a real or imaginary axis” (Mitchelmore & White, 1998, p. 7), highlights a dynamic nature (Keiser, 2004; Mitchelmore & White, 2000). This represents a turning movement from one position to another where the amount of turn determines the size of the angle formed between the two positions when the rotation is less than 360° (Mitchelmore & White, 1998).

Given the multifaceted nature of the concept of angle, questions emerge regarding the extent to which students understand angles. Researchers studied the students’ angle understanding in different contexts, such as how they define angles (Erbay, 2016; Keiser, 2000, 2004), how they construct different angle images such as 0-line (i.e., a 360-degree angle), one-line (i.e., a 180-degree angle) and two-line angles (Mullins, 2020), and how they form the concept by integrating various angle contexts such as slope, turn, and intersection (Mitchelmore & White, 1998, 2000). In defining angles, Erbay (2016) found that sixth graders mostly used the term *two rays* for defining angles and only a few students stated *one common point* or *the region between two rays* in their description. Keiser (2000) observed that students excluded some aspects of angles such as the turn aspect. Researchers observed that students had difficulty imagining the angles 0°, 180°, and 360°, to relate the amount of turn with the angle concept (e.g., Erbay, 2016; Keiser, 2004; Mitchelmore, 1998). This difficulty was observed even in pre-service mathematics teachers (e.g., Yiğit, 2014). Mullins (2020) indicated that difficulty in recognizing 0°, 180°, and 360° angles stems from a tendency to focus on a static view of angles. However, she also noted that students with more “fluid” thinking (p. 39) can integrate and transition between the static and dynamic aspects of angles as they develop their understanding of the concept.

Furthermore, researchers specified some student errors and misconceptions about angles, which indicates a limited understanding of the concept (e.g., Bütüner & Filiz, 2017; Clements, 2003; Devichi & Munier, 2013). Some prevalent misconceptions were reported, such as believing that the size of rays of angles depends on the length of arms/rays (Clements, 2003; Mitchelmore, 1998; Stavy & Tirosh, 2000) and thinking that the larger radius of an arc which is representing an angle means a bigger angle (Bütüner & Filiz, 2017; Mitchelmore, 1998).

Students’ limited understanding of the angle concept and reported misconceptions about the subject may represent the lack of viability of the concept, indicating that it is not yet valid in certain mathematical contexts and subjects (von Glasersfeld, 1981). For example, students who have constructed angle knowledge only through an intersection of two lines may struggle to maintain the meaning in rotational contexts such as 0-line angles representing a 360-degree angle through a whole rotation (Mullins, 2020). Therefore, we thought that investigating students’ knowledge of angles, including their current schema, conceptions, and level of abstraction, would reveal the gap between a more comprehensive outcome for angle conceptions and students’ actual conceptions. Keiser (2000) supported a comprehensive and flexible understanding of the abstract concept of angle, encompassing the various meanings and contexts that allow students to adapt the concept in different situations. Therefore, in this study, we aimed to investigate students’ conceptualization of angles in terms of how they interpret angles by connecting them within different contexts.

Theoretical and Conceptual Framework

In this study, we draw on two main theoretical and conceptual frameworks. First, radical constructivism (Piaget, 2001; von Glasersfeld, 1984) shaped our understanding of knowledge construction and guided the formulation of our research questions, as well as the analysis and interpretation of data about the concept of angle. Second, the conceptual framework of Mitchelmore and White (2000), which outlined the stages of angle knowledge development through the constructivist perspective, informed the design of our data collection and the subsequent data analysis. The following paragraphs clarify how these perspectives shaped various aspects of our study.

According to radical constructivism, students learn mathematical concepts by abstracting them through mental operations supported by experiential situations (von Glasersfeld, 1996). For instance, counting as an experiential activity initiates the students' development of the number concept, further enabling them to perform more advanced operations with numbers without relying on sensory materials. Similarly, when constructing the concept of angle, students are expected to begin with experiencing physical angle situations such as opening doors, rotation of the clock hands and climbing (Casas-Garcia & Luengo-Gonzalez, 2013; Munier et al., 2008).

Gradually, they would construct an abstract understanding of the angle concept by connecting it with different contexts and formal definitions by reflecting on the concept in their mind (Mitchelmore & White, 1998). In this process of constructing mathematical concepts, students use the outputs of previous learning experiences to reorganize them and transfer them to higher levels of structures, which is called reflective abstraction (Piaget, 2001). In reflective abstraction, the connections between the mathematical concept and different mathematical ideas or contexts determine the level of abstraction (Mitchelmore & White, 1995). More connections refer to a higher level of abstraction, indicating "a deeper understanding of mathematical concepts" (p. 65).

In the case of constructing an abstract and comprehensive concept of angle, we draw on the perspectives of Mitchelmore and White (2000) and Mitchelmore (1997) explained how students possess angle knowledge and re/construct the angle concept through reflective abstraction. Mitchelmore and White (2000) proposed three stages in terms of students' construction of the angle concept, which are hierarchical and also overlapping: *situated angle concepts*, *contextual angle concepts*, and *abstract angle concepts*. Situated angle concepts include a class of similar experiences that determines a specific angle situation for a child. For example, recognizing a hill by interpreting the steepness through walking down and up is an example of the situated angle conception. Contextual angle concepts include a context with similar angle situations in which a child recognizes the relations between different angle models, such as connecting different situations including slope (i.e., hill, ramp, roof) or turn (i.e., wheel, fan, door). Lastly, abstract angle concepts appear by bringing different contexts together, considering their similarities regarding a particular angle domain. To illustrate, a child who can recognize the corner of a tile as fitting into the space on the slope of a ramp constructs an abstract angle concept by relating the contexts, slope, and corner. Therefore, in this study, by the term 'a comprehensive and abstract angle concept' we refer to an understanding of angle including the interpretation and connection of different aspects of angles such as turn, space, and intersection. In this way, we adopt an understanding of angles that accommodates various angle situations within the formal definition of *two rays with a common endpoint* (Keiser, 2000), such as the space formed by the rotation of a ray from one position to another or between two rays intersecting at a common endpoint.

The Rationale and the Purpose of the Study

Studies on students' conceptions of angles are rare, with existing research taking various approaches, such as examining students' descriptions of angles following informal geometry instruction (Keiser, 2004; Visher, 2020), comparing angle schemes across different grade levels (Mullins, 2020), and investigating how students recognize angles in diverse contexts by relating them to the standard angle concept (Mitchelmore & White, 1998, 2000). Generally, researchers characterize angles as abstract yet common in everyday life (Tanguay & Venant, 2016), with students expressing and understanding them in various ways and contexts (Keiser, 2004; Mitchelmore & White, 2000; Mullins, 2020). A review of recent national studies reveals a predominant focus on students' misconceptions (e.g., Bütüner & Filiz, 2017; Ozen-Unal & Urun, 2021) or their learning competencies related to angles within the curriculum (e.g., Erbay, 2016). However, the depth of students' conceptualization of angles, as an abstract and multifaceted concept (Keiser, 2004; Tanguay & Venant, 2016), which can be developed through interaction with various contexts (Mitchelmore & White, 2000; Mullins, 2020; Visher, 2020), has not been fully explored. In this study, we aimed to address this gap by investigating the students' conceptualization of angles through a constructivist approach, offering a more comprehensive understanding of how students conceptualize angles.

We adopted a constructivist perspective because it advocates interpreting students' knowledge through the interrelationship between existing concepts and how they fit into that structure, rather than the correspondence between students' knowledge and the objective reality (von Glasersfeld, 1984). This approach is particularly relevant to the nature of the concept of angle, which is not fully addressed in national studies of angle conceptualization. In constructivism, students' abstraction of concepts and their viability gains importance, in which they construct the connections between the existing structures and experiences of the concept. In our study, we interpreted the abstraction of angles like Mitchelmore and White (2000) through the connections between different contexts of angles, such as intersection, turn, and slope. To investigate students' abstraction, we worked with students who have already experienced many physical situations and encountered examples of angles in school mathematics.

The concept of angle is generally introduced in the early years of elementary school (e.g., National Council of Teachers of Mathematics [NCTM], 2000; Turkish Ministry of National Education [MoNE], 2018). For example, according to the Common Core State Standards (CCSS, 2010), second graders are expected to compare geometric shapes through their angles and other characteristics. However, the concept of angle as a geometric shape and its definition is introduced in the fourth grade by representing the angle through circles, turns, and degrees. In the current Turkish mathematics curriculum, the angle concept appeared first in the third grade through real-life objects such as open scissors, the space between hour and minute hands of a clock. In the fourth-grade, students are expected to understand that an angle is formed by rotating a ray around its starting point, using a compass. The formal definition of angles (i.e., two rays with an intersecting corner) was first introduced in the sixth grade (MoNE, 2018). Given the sixth graders' prior knowledge of angles based on the curriculum and their developmental stage, which aligns with Piaget's formal operational stage, we aimed to explore their conceptions of angles. We conjectured that they would develop an abstract concept of angle by constructing connections with different angle situations.

In this way, we believe that investigating the conceptualization of the angle concept, which includes multi-faceted definitions in the previous studies (e.g., Keiser, 2004; Mullins, 2020) would provide valuable information for teachers and teacher educators to improve instruction and curriculum. In parallel with this aim, the study addressed the following research question and its corresponding sub-questions:

1. How do sixth-grade students conceptualize the angle concept in different contexts such as slope, turn and openness?
    i. What aspects of angles do the sixth-grade students' descriptions include?
    ii. How do the sixth-grade students identify and interpret angles within different contexts, such as the slope, turn, and openness?

Method

In the current study, a qualitative case study was employed to examine a particular case, group, or phenomenon in depth (Yin, 2003). The sixth graders' conceptualization is the case of this study, as a particular group. The age of sixth graders represents a process when their mental operations are ready to construct abstract mathematical concepts. In addition, up to this grade level, students have learned about the angle concept through real-life examples, the elements of geometric shapes as well as the formal definition.

Participants and the Context

This study was conducted with eight sixth-grade students (11-12 years old) from two different classrooms in a public middle school in İstanbul, Turkiye. They were selected based on both purposeful and convenient criteria. The selection of the school was convenient due to easy access to the school principal, parents and students, and obtaining permissions for data collection. The school was a small middle school in the neighborhood with a moderate socioeconomic status.

The grade level was purposefully selected to align with students' prior knowledge and experience with the concept of angle, as outlined in the curriculum, and their readiness for the formal operational stage according to Piaget's theory of cognitive development. The Ministry of Turkish National Education's mathematics curriculum (MoNE, 2018) outlines learning objectives related to angles in the grades leading up to sixth (e.g., 4th and 5th), where the concept was introduced in various physical contexts, and that the formal definition of angles is introduced in the sixth grade. Therefore, we assumed that this equips sixth graders with the knowledge and experience with the concept of angle that is needed for their conceptualization of angle through new connections and abstractions. Additionally, we hypothesized that sixth graders would recognize and interpret angles in different contexts, such as the slope, turn, and openness through connections since the students' age aligns with the transition from the concrete operational stage to the formal operational stage.

The students attended the study voluntarily. In addition, we intended to generate a heterogeneous group of participants as a sample representing a regular sixth-grade classroom including students from different achievement levels. Therefore, among the 18 volunteer sixth graders, the first author, who is also the students' mathematics teacher, purposefully selected students from different achievement levels by dividing the class into four groups: high, high-moderate, moderate, and moderate-low based on their previous math exam scores in the last two academic years. For instance, students with exam scores of 85 and above out of 100 were considered high achievers, while those with scores between 75 and 85 were classified as high-moderate, and so on. The math exams conducted at the school evaluated students' achievement of the current learning objectives. By selecting students from various achievement levels, we aimed to ensure that the conceptualization of angles was independent of mathematics achievement and free from bias. We also aimed to select students who are good at expressing themselves to get richer data based on the first author's individual observations in teaching. At the end, there were two students in each achievement level; four were female, and the other half were male.

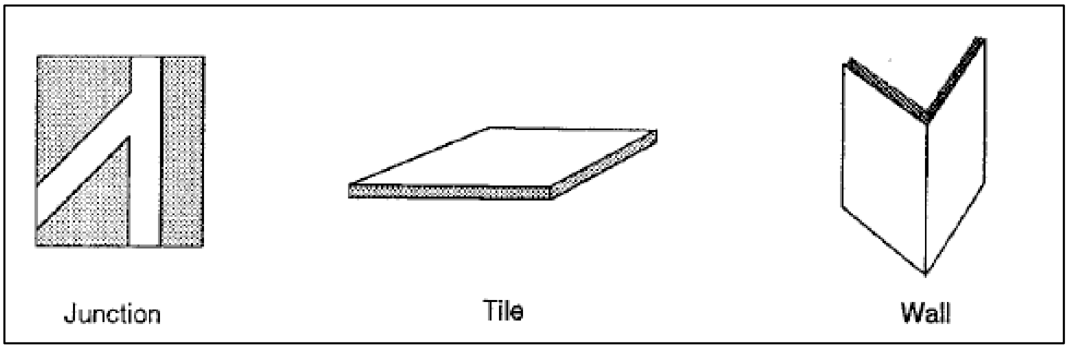
Data Collection Tools

We investigated students' conceptualization of angles in two parts through a structured interview consisting of four questions (see Appendix). To reveal students' understanding of the angle concept, Keiser et al. (2003) focused on how students defined angles, the elements they highlighted, the words they preferred, or their reference to rays, interior, or turn. Considering this, in the first part, we asked the students to define angles and describe the elements forming an angle. They were also asked to draw an angle by considering the case of being unable to put it into words during the interview and to see the picture of the concept in their minds. In addition, we asked them to give real-life examples of the angle concept to see with which aspects they can identify the concept, such as intersection (static), space, and turn (dynamic). In this way, it is aimed to understand the extent of the connections the students can construct in developing the angle concept regarding its different meanings and being static or dynamic.

In the second part, the aim was to explore in which contexts and situations the students can recognize and explain angles, such as the slope of a ramp, the turn of a door, corners of things, circular shapes as a whole turn, and 0-line angles. In this part, the students were shown twelve pictures demonstrating different angle situations, such as ramps, roofs, circles, scissors, an opening of a book, and a door (see Appendix), which were adapted from the study of Mitchelmore and White (2000) who utilized physical models of angle situations to uncover students' concept formation. The pictures in the study of Mitchelmore and White (2000) demonstrated three common contexts, corner, turn and slope, which also have different characteristics such as being movable or fixed and having a turn or open situation. For example, they used junction, tile and wall models (see Figure 1) to represent a fixed angle scenario which students most commonly interpreted as standard angle models.

Figure 1

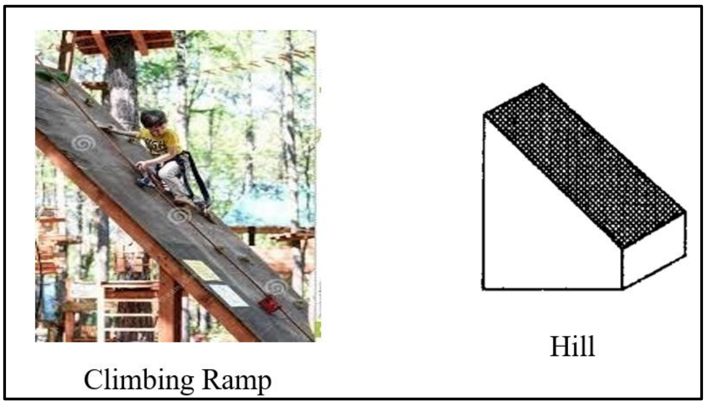
Pictures for Fixed Angle Situations in Mitchelmore and White's Study (2000, p. 221)



In a similar approach, we used images of a roof, paving stones, and a crossroad to represent the intersection and fixed-angle situations (see Appendix). Furthermore, Mitchelmore and White (2000) had a wheel, a door, scissors and a hand fan picture to depict movable situations involving turning and opening movements. In our study, we employed images of a wheel, a door, a wall clock, and a circle (see Appendix) to examine whether students could link the angle concept with a turning movement. Additionally, in the image of a circle (see picture E, Appendix) we aimed to examine how students recognize a 0-line angle by associating it with a full turn (Keiser, 2004; Yiğit, 2014). Lastly, we used a traced figure of a ramp and pictures of a climbing ramp and a sloping road to include the slope context which was utilized by Mitchelmore and White (2000) through the pictures of a hill and a signpost. To differentiate the ramp figures from an intersection context and to highlight the slope context, we intentionally omitted the ground as a horizontal plane in the picture of a climbing ramp (see Figure 2, left).

Figure 2

The Pictures Depicting the Slope Context in the Current Study (left) and in the Study of Mitchelmore and White (right) (2000, p. 221)



Mitchelmore and White (2000) emphasized that the absence of the horizontal arm in a sloping line causes difficulty for students to associate it with a standard angle concept (see Figure 2, right). Therefore, we wanted to challenge the students to imagine a horizontal plane and associate the slope with a standard angle concept. It is important to note that the pictures do not solely represent one specific context. For example, an image highlighting a turn context, like the wheel picture, could also be interpreted as representing angles in the intersections of the wheel's spokes rather than its rotational movement.

Keiser (2004) reported that students can have difficulty identifying angles that lack a vertex and two different arms. In the last part of the interview, we presented students with one-ray model and asked whether they could identify any angle. Our aim was to observe whether they could recognize angles of 0 or 360-degree angles in this one-ray context.

Data Collection Process

For the data collection process, ethical approval was obtained from the Institutional Review Board of the Ethics Committee. The parents provided consent for their children's participation by signing consent forms and receiving detailed information about the research. They were informed about the study's objectives, as well as the issues of trustworthiness, confidentiality, and anonymity. Parents were assured that participants' identities and personal information would remain confidential, that records would be securely stored, and that pseudonyms, rather than real names, would be used in the study. They agreed to their children's participation under those conditions.

The main data source of this study was the interviews. The interview protocol was conducted in an online environment through the Zoom video conferencing platform. Each interview took 15 to 30 minutes and was video recorded. The questions were shared through screen sharing one by one, and the students could draw something on the screen when required. Four people were at the conference in each interview: three researchers (the first three authors) and one student. The first author asked the questions, and the other two researchers observed the interview process by taking notes and also asked questions when they needed further explanations from the students.

Data Analysis

The data analysis part started with the ongoing interviews. We took notes during interviews and discussed the students' answers after each interview. We, all authors, conducted an initial data analysis by writing analytical memos and transcribing the important responses during the interviews. After each interview, we discussed the entire interview process for each student to reflect our analysis of the student's conceptualization of angles and to enhance data reliability through triangulation.

In the data analysis process, we adopted the constant comparison method, which involved the comparison of data of the same question from different students and from the same students for different questions (Charmaz, 2005). Initially, the first three authors independently followed this comparative process through open coding and by extracting codes directly from the literature, as detailed in the next paragraph. Subsequently, the researchers convened to compare their codes using the constant comparative method. We discussed the similarities and discrepancies among the codes to reach a consensus on the students' conceptions of angles and to generate our themes. This final step also contributed to ensuring the credibility and trustworthiness of the study (Savin-Baden & Major, 2013).

We will now explain how we adopted the codes from the literature during our initial coding process, in addition to open coding. For this purpose, we documented the terminology used in the literature, that clarifies the multifaceted concept of "angle," including terms such as intersection, corner, static, dynamic, slope, and turn (Keiser, 2004; Mitchelmore & White, 2000). For instance, a student's definition of angles as the intersection of two rays was assigned the codes "static" and intersection." As the analysis progressed and further data was reviewed, the student's interpretation of angles in the context of a ramp image led to a new code, "slope context," reflecting a broader conceptualization of angles. Sample codes, example answers and categories are represented in Table 1.

Table 1

Example Responses and Corresponding Codes in the Data Analysis Process

Example Responses	Corresponding Codes
Angle between arms	Physical experiences
Angles while opening window/door	Openness
	Dynamic
Angle in the corners	Corner
	Static
Angle in the sloping road	Connecting slope and angle
The difference between the horizontal and sloping road	Abstraction

The students' definitions, statements of angle elements, and drawing angles were analyzed to gain insight into their conceptualization in terms of their descriptions of angles. We evaluated their descriptions in terms of how comprehensive they are by including different aspects such as turn and openness and whether they are both dynamic and static definitions. This analysis generated the first theme of our findings. In addition, we coded the students' real-life examples for angles and how they identify angles within different contexts such as turn, slope, space, and intersection (Mitchelmore & White, 2000). For example, a student who identified angles in corners or intersecting figures, as convenient to the formal definition, represents that their angle knowledge sustains in the context of intersection. In addition to this, if the student also recognized an angle in the sloped road figure (i.e., between the road and the horizontal axis), revealed a connection between the angle concept and the slope by reflecting on prior angle knowledge. We interpreted this as a broader and more abstract conceptualization of angles in terms of constructing more connections between angles and different contexts. These interpretations produced our second theme.

Findings

This part presents the analysis of students' conceptualization of angles into two main sections parallel to the research questions which are i) students' angle descriptions and ii) students' interpretation of angles in different contexts. The analysis of how they defined the concept verbally, identified the elements of angles and drew angle figures represented only their description as an aspect of their conceptualization. Further, their recognition of angles in real-life situations and interpretation of different contextual figures such as wheels, slopes, and corners revealed another aspect of their conceptualization. Overall, this analysis gave a thorough picture of their understanding of angles.

Students' Conceptualization of Angles: Angle Descriptions

The analysis of students' definitions of angles and their description of angle components showed that their conceptualization was primarily based on the formal definition (i.e., angles generated by two rays with an intersection), indicating a static meaning. Most of the students' descriptions involved two main components: an intersection (or corner) and rays (or segments). However, their angle descriptions differed in terms of comprehensiveness, accuracy, and connection to related constructs.

Half of the students could accurately define the angle concept similar to the static definition. Three examples of students' definitions are the following:

Ali: A shape formed by rays and having one corner.

Hakan: Angle of the point where two line-segments meet.

Mert: Formed by two rays, an interval between two rays having a common point.

As seen in the responses, their descriptions included important angle components such as rays (or segments) and corners (i.e., intersection or common point). However, Mert gave the most comprehensive definition by including a space meaning of angles through the expression "interval between two rays". He expanded the concept of angle beyond its intersection meaning by incorporating the idea of space, giving the concept a measurable entity in addition to a geometrical shape. For this reason, we evaluated him as having constructed more connections between the angle concept and the related contexts, resulting in a more comprehensive conceptualization.

We also observed that a student defined angles by using only the related concepts, such as degree and corner, and real-life examples that have a salient corner image as follows:

Rana: An angle is something that depends on the degree of a place, for example, a corner to a corner of a bookcase is an angle.

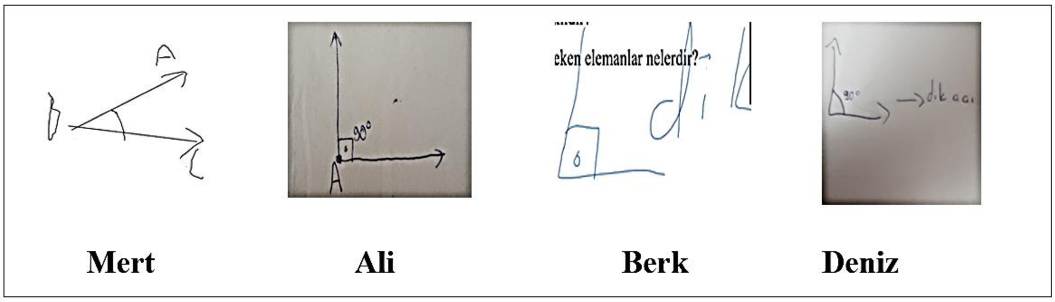
Although Rana was unable to provide a precise definition of an angle, she was still able to identify its components, such as rays, corners, and intersections. She stated the proper angle components as being "two rays/sides and one joining point". It may show that her conceptualization mirrors the static definition of angles and common right-angle examples from real life including objects with a right corner. Her example of corners from real life demonstrates the static way of her angle knowledge too.

Lastly, the other three students did not include the main angle components from their textbooks in definitions of angles. Instead, they used related terms such as degree, intersection, and surfaces, or gave examples of angle types. For instance, Selin defined angels as "Two intersecting surfaces" and identified the components as "numbers and lines," focusing on the intersection aspect. She considered the intersection aspect of the angle concept, which carries a static meaning. Similarly, Deniz described angles as "Something with degrees, there are angles with 90 degrees, acute angles, and obtuse angles" and listed "degree and shape" as components. It seems that her angle image is limited to angle types based on degrees, as taught in earlier grades (MoNE, 2018). Hence, her conceptualization of angles refers to geometrical shapes.

As seen in previous examples, most students' descriptions included terms such as corner, intersection, and joining points. However, several students did not provide an accurate definition of angles or mention the components. This does not necessarily indicate a less comprehensive conceptualization. It may reflect their ability to express their understanding verbally, which might not fully reflect their overall conceptualization. Regarding this, we also asked them to draw angle figures to enrich our understanding of their angle conceptions. Although the students varied in their verbal definitions of angles, the models they drew were very similar. All students drew a complete angle figure with a corner and rays/segments which is consistent with their angle descriptions (see Figure 3). The angle figures included key elements such as rays, intersections, and corners, which are commonly represented in classroom settings. Therefore, the students who struggled to provide a proper verbal definition were still able to demonstrate their angle image through their drawings in this part. This also informs us that the students' conception of angles may be more comprehensive than their verbal descriptions.

Figure 3

The Examples from the Students' Figural Representations of Angles



Students' Conceptualization of Angles: Extending the Concept into Different Contexts

In addition to the students' descriptions of angles, they were asked to give real-life situations of angles and were given a table of contextual figures to identify angles (see Appendix, Question 3). How they recognized angles in real-life situations and various contexts provided valuable insights into the scenarios where they could make connections to angles, such as sloping surfaces, circles, and objects with salient corners.

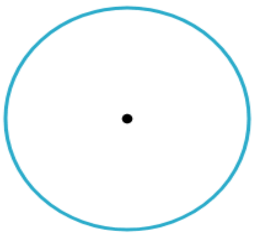
The sixth graders in this study consistently looked for an angle shape with apparent two arms, an intersection, or a corner when giving real-life examples or interpreting various figures. Specifically, all students identified the corner aspect of angles by pointing to the corners of objects in perpendicular shapes, such as doors, books, tables, and TVs, which aligned with their verbal definitions of angles. Additionally, they were quick to recognize angles in images that included apparent corners, space, and intersecting segments, mirroring their real-life examples. For instance, in the roof figure, all students immediately showed the corner at the top as representing an angle. Ali simply stated, "There exist angles in all shapes with corners," and similarly, Deniz mentioned only the corners of rectangular objects.

Some students provided more comprehensive responses than their verbal descriptions by addressing the space aspect of angles in real-life examples, such as body movements, opening windows and doors, and the space between the pages of an open book. For example, Selin said: "When we open or close scissors, an angle occurs." It is important to note that the given answers reflected prototypical instances commonly found in textbooks or classroom settings, which the students were likely familiar with. Therefore, these examples, including the space or openness meaning of angles, may have been recalled from prior experiences rather than constructed through a direct connection to their own understanding.

The students' conceptualization of angles did not include the idea of turn or one-ray angles, as evidenced by their interpretation of circular shapes and one-ray figures. For instance, in a wheel image, the students focused on the intersections of wheel's spokes to find an angle, rather than recognizing a 360-degree angle at the center of the wheel. Furthermore, none of the students identified an angle in a circle image (see Figure 4); instead, they looked for a corner to show the presence of an angle. In response to the circle figure, five students stated that there were no segments or rays, and therefore no angle in the circle. One student (Berk) explained, "The angle is not a round shape, so there is no angle, our teacher did not show there is an angle in circles before." Similarly, Ali stated that there is no corner in the circle.

Figure 4

Picture E Shown to the Students in the Interview

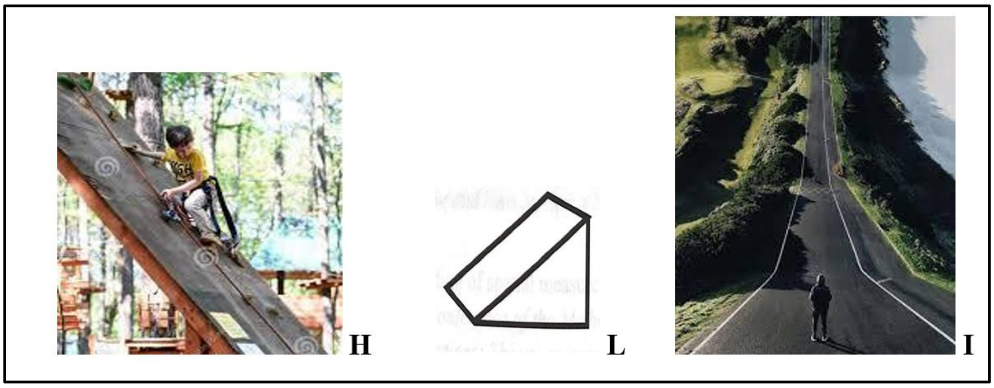


Although the students had previously provided real-life examples of turning movements (e.g., opening arms and doors) to illustrate angles, they could not recognize or generate a 360-degree angle in a circle based on the concept of turning. Similarly, they could not identify a 360-degree angle in a one-ray model. When asked, the students stated that a single ray was not enough to generate an angle (Question 4) and attempted to draw another ray with a common endpoint to generate an angle. These show that they had not fully grasped the turning aspect of angles. Only one student, Mert, initially considered the one-ray figure as a 360-degree angle, but he later changed his response, saying "Only one ray is not enough; another ray must be drawn to generate an angle."

Furthermore, the students' ability to recognize an angle in pictures including ramps, inclines, and slopes, revealed whether they could relate the concept of slope to angles as another aspect. Only three students (Ali, Mert, Selin) identified an angle between the ramp and the ground in picture H (see Figure 5). Other students primarily focused on identifying salient intersecting segments that formed an angle, such as the lines on the ramp or the legs and arms of the boy (see Figure 5, picture H), rather than recognizing the space between the ground and the inclined wood. Interestingly, those who identified angles in the ramp in picture L (see Figure 5) failed to notice any angle in the wooden ramp, a real-life image with a similar context. Picture L appears more like a sketched figure found in textbooks, with apparent segments and corners, while picture H requires more interpretation and abstraction by the meaning of angle. This might have caused the students to have difficulty in recognizing angles in picture H.

Figure 5

Pictures from the Interview Representing Different Situations of Angles



Similarly, in another figure including slope (see Figure 5, picture I), in the road picture, only one student, Mert, identified an angle by referring to the slope of the road. He said, "There can be an angle between the inclination and the flat road but normally there is not an angle on a road. There cannot be an angle in a flat road." Other students who identified angles in the road picture focused on the angles between the road lines rather than considering the surfaces. This may show that an angle in the context of slope was not explicitly present in the sixth graders' conceptualization of angles in this study. However, Mert demonstrated a more abstract understanding of angles by reflecting on the meaning of angles to connect it with the slope of the road. This also highlighted his abstraction of the angle concept through the understanding of the space aspect of angles, which he had also referenced in his definition.

Discussion and Conclusion

In this study, we investigated sixth graders' conceptualization of angles, focusing on their verbal descriptions, figural representations and their ability to identify angles in various contexts such as slope, turn, and openness. The findings revealed that the students' conception of angles mainly included the static components of angles and reflected the standard angle definition (i.e., the intersection of two rays/segments) through their emphasis on the corners and the intersecting rays or segments. This finding is consistent with other studies that have examined students' understanding of angles (e.g., Erbay, 2016; Mitchelmore & White, 2000; Ozen- Unal & Urun, 2021; Visher, 2020). Researchers reported that during elementary school, students generally demonstrate a standard angle understanding formed by the corner and intersection contexts (e.g., Mitchelmore & White, 2000; Tanguay & Venant, 2016). In our study, students incorporated a broader range of aspects only when providing real-life examples. For instance, they included the concept of openness through examples like body movements and the concept of turn through examples such as opening door. However, their conceptualization generally lacked connections to other contexts such as slope and rotation, as seen in their interpretations of various angle situations in figures. For example, their failure to recognize an angle in a circle figure and to identify angles in a sloping road figure demonstrated their understanding of angles with a lack of connections to multiple aspects of angles.

In terms of verbal descriptions of angles, Mert had the most comprehensive angle definition amongst the other students, including the aspects of both intersection and openness. Additionally, other students had difficulty defining the concept in words, as seen in other studies (e.g., Erbay, 2016; Ozen- Unal & Urun, 2021) while their drawings of angles were more comprehensive than their verbal descriptions regarding the components of angles such as rays, corners, and intersections. This discrepancy may be attributed to the students' difficulty expressing or explaining concepts verbally. Ozen-Unal and Urun (2021) found that while some students initially provided a narrow definition of an angle, they were open to accepting other definitions when they were presented to them. Therefore, this discrepancy may represent that they have a concept image for angles, yet they have "inactive or forgotten" concept definition (Vinner & Hershkowitz, 1980, p. 179). This indicates that while students may have a broader conceptual understanding of angles, they may forget the formal definition or specific terminology such as *ray* and *segment* when describing it verbally.

Similar to their descriptions of angles, the students' identification of angles in various figures was limited to the image of an intersection and obvious two arms. For example, most of the students could not explicitly state a turning aspect of angles as parallel to other angle studies examining the angle conceptions of middle school students (e.g., Bütüner & Filiz, 2017; Erbay, 2016; Keiser, 2004; Mullins, 2020). When the students could not see any point and space between two rays in the circle picture (see Figure 2), they indicated that there was no angle, thereby excluding the meaning of turn from their interpretation. According to Mullins (2020), recognizing an angle in a circle model may indicate that students possess knowledge of angles, including the aspects of rotation or turn. Although the students mentioned body movements and opening doors as examples of angles, indicating its dynamic nature (Clements & Burns, 2000; Keiser, 2004; Mitchelmore, 1998; Smith et al., 2014), this does not necessarily imply that they fully assimilated the turn and space aspects of angles. Since these examples are common in textbooks, it is possible that the students learned them by rote, rather than engaging with the more comprehensive angle conception that incorporates both the space and turn aspects.

The findings also showed that the students could not recognize one-ray angles, consistent with previous studies (e.g., Keiser, 2004; Mullins, 2020). Mitchelmore and White (1998) noted that it would be hard to identify 0-degree or 360-degree angles when angles are conceptualized only as the region between two rays. Similarly, Bütüner and Filiz (2017) observed that students faced the same difficulty with one-ray or one-line angles. They attributed this difficulty to the static angle definitions found in textbooks, which limit students understanding of more dynamic or context-based representations of angles. As a result, students do not consider rotating the ray to construct an angle, which reflects a lack of understanding of the dynamic aspect of angles (Keiser, 2004; Mullins, 2020).

In this study, the students' limited conceptualization of angles appeared also in their interpretation of the images including sloping surfaces. Most students could not recognize the angle formed between the ramp (the sloping surface) and the ground (the horizontal surface) (see Figure 3, picture H). This finding aligns with Mitchelmore and White's (1998) observation that sloping surfaces/planes were less likely to be recognized as forming angles compared to sloping lines. The students appeared unable to expand their conception of angles beyond the intersection of lines, failing to include the intersection of surfaces, which requires a reflective abstraction. This showed that the context of slope does not appear explicitly in sixth graders' angle conception in this study. However, it is noteworthy that one sixth-grade student, Mert, could reflect on the formal definition of an angle and recognize other aspects and meanings of the concept. For instance, he could identify the angle between the horizontal plane and the ground plane in a road figure, demonstrating a more advanced understanding of angles. He connected the idea of angles to the concept of slope, by reflecting on the meaning and definition of angles and he could state a more comprehensive angle description by indicating both the intersection and the space between the rays. His thinking and responses during the interview demonstrated that he could structure a more developed abstract knowledge of angles by reflecting on the assimilated description of angles (von Glasersfeld, 1995).

In conclusion, students' description of angles and their interpretation of angles in various contexts revealed that their angle knowledge is viable only in contexts including a salient intersection and two rays, consistent with the formal definition. This finding highlights the note of Keiser (2004) about how introducing formal angle definition may limit elementary students' understanding of angles. She noted the importance of introducing the angle concept through its different meanings before the formal definition is introduced so that students can identify a similarity between different angle contexts, such as relating a sloping surface or a rotation/opening situation with a corner or bend, indicating an abstract angle conception (Mitchelmore & White, 2000). Although researchers observed that, during elementary school, many students demonstrated a standard angle conception including corners and intersection (e.g., Mitchelmore & White, 2000; Tanguay & Venant, 2016), they also observed that there are students, even at the beginning of elementary school (Mitchelmore & White, 2000), who recognized other angle contexts such as the space meaning of angles by referring to a measured object in degrees (Tanguay & Venant, 2016). The sixth graders in this study, except Mert, did not demonstrate this way of conception of angles. According to the Turkish Mathematics curriculum (MoNE, 2018), sixth graders have received instruction on angles starting from primary school, which included different aspects and real-life situations in addition to learning the formal definition of angles. Therefore, we primarily expected the sixth graders to construct these connections between different aspects of angles relying on the previous studies and the content of the curriculum. However, the students' limited identification of angles in various contexts (e.g., slope, turn, space) without flexibly linking multiple angle contexts demonstrates their lack of abstraction and the lack of viability of the concept in different situations.

Implications, Limitations and Recommendations for Research

The findings of this study have several practical and theoretical implications. With respect to the practical implications, the findings inform mathematics educators and curriculum developers about the missing aspects of the multifaceted angle concept in sixth-grade students' conceptions such as turn, and openness. This can provide valuable insights for mathematics educators to improve their approach in teacher education programs in relation to introducing the angle concept through the discussions of its different meanings. Consequently, teachers who know the multi-faceted nature of angles and students' lack of understanding may place greater emphasis on exploring various aspects and contexts of angles in their teaching. For example, a static angle figure can be animated by rotating a ray-like object or opening items to form a space between them, helping students to relate different aspects of angles. Keiser (2000) noted that students' exposure to formal definitions without sufficient engagement in different physical situations of angles could "prohibit students from further exploration of this abstract concept." (p. 510). It is worth noting that the new Turkish mathematics curriculum (MoNE, 2024), which will be implemented gradually in the 2024-2025 school year, includes the turn aspect of angles starting from the fourth grade by integrating measurement activities and real-life examples. Providing examples of a concept as an initial experience is crucial for students' first concept formation (Vinner, 2018; Zandieh & Rasmussen, 2010). Therefore, it is recommended to continue providing real-life and contextual examples of angles in all grade levels, covering all aspects, to help their abstraction of angles.

Regarding theoretical implications, our research offers valuable insights into how sixth-grade students develop a more abstract understanding of angles. Despite receiving the same instruction on angles, in this study the students' conceptualization differed, likely due to variations in cognitive development or other factors. Specifically, one student was able to relate the angle concept to the context of slope by reflecting on its formal definition while others struggled to make similar connections. This suggests that an abstraction of angle is achievable at this grade level. However, this was only observed in one out of eight students, indicating that this level of abstraction might not be readily accessible to all students. This diversity may reflect either a lack of reflection or insufficient assimilation of angle description. To address this, we recommend incorporating teaching processes that encourage discussion, allowing students to reflect on their prior knowledge and develop a deeper understanding of angle concepts.

Our study had the limitation of including only eight participants. Future research may increase the number of participants and work with students from different backgrounds to have substantial data in terms of angle conceptualization. This would enrich the data concerning diversity in students' conceptualization of angles. Additionally, we recommend conducting similar studies with students from other middle school grade levels to produce a more comprehensive understanding of middle school students' conceptualization of angles and the impact of grade/age level on their conceptualization.

Code of Ethics

Ethical approval and written permission were obtained from Middle East Technical University Applied Ethics Research Center (Application No: 0034-ODTUİAEK-2023, Approval Date: January 27, 2023)

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Altıncı Sınıf Öğrencilerinin Açı Kavramını Anlamlandırması

Öz

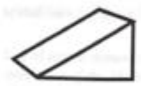
Bu çalışmanın amacı altıncı sınıf öğrencilerinin açı kavramını nasıl anlamlandırdıklarını incelemektir. Bu amaçla, sekiz tane altıncı sınıf öğrencisiyle açı kavramını sözel olarak nasıl ifade ettikleri ve eğim, dönüş ve açıklık gibi gerçek yaşam bağlamlarında nasıl yorumladıklarını incelemek için birebir görüşmeler yapılmıştır. Bulgular, öğrencilerin açı tanımlamalarının çoğunlukla standart açı tanımında olduğu gibi köşe ve kesişim gibi açının statik bileşenlerini içerdiğini göstermiştir. Sadece bir öğrenci, standart açı tanımından yola çıkarak açıları eğim ve aralık bağlamlarıyla ilişkilendirerek daha kapsamlı bir anlayış sergilemiştir. İlköğretim müfredatında açı kavramının gerçek hayattan örneklerle ve fiziksel deneyimlerle yer almasına rağmen öğrencilerin çoğunun eğim ve dönme gibi farklı bağlamları açılarla ilişkilendirmekte zorlandığı bulgusu eğitimle ilgili çıkarımlar açısından tartışılmıştır. Ayrıca, açıların çoklu temsilleri ve bağlamlarının sınıf içinde soyutlamaya imkân vererek sunulmasının önemi vurgulanmaktadır.

Anahtar kelimeler: matematik eğitimi, ortaokul, altıncı sınıf, açı kavramı, kavramsallaştırma

Appendix

Interview Questions

1. **What is an angle? Can you define it?**
    - What do you understand by the term of angle?
    - What are the components of an angle?
    - Can you express the elements of an angle that are necessary to form an angle?
    - Can you draw an angle?
2. Can you give an example of an angle from daily life? Which situations can be an example of an angle?
    - Why do you think so?
3. Which of the following figures might be an example of an angle? Why?

A  Wheel	B  Roof	C  Clock
D  Paving Stone	E  Circle	F  Door
G  Scissors	H  Climbing Ramp	I  Road
J  Crossroad	K  Book	L  Ramp

4. Is it possible to generate an angle by using the given ray below?