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RESEARCH ARTICLE

Chasing Volatility of USD/TRY Foreign Exchange Rate: The Comparison of CARR, EWMA, and GARCH Models

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Abstract

This paper aims to make a comparison between range-based and return-based volatility models. For this purpose, we compare the Conditional Autoregressive Range (CARR) type and Generalized Autoregressive Conditional Heteroskedastic (GARCH) type models with different innovation distributions and the Exponential Weighted Moving Average (EWMA) model with fixed and estimated lambda parameters. The out-of-sample forecasts obtained from the volatility processes are compared according to the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Heteroskedastic Root Mean Square Error (HRMSE), and Heteroskedastic Mean Absolute Error (HMAE) statistics. We use the USD-TRY exchange rate data for real-life applications since estimating the volatility of forex helps to determine prices for goods and services to avoid the uncertainty created by exchange rate shocks in developing countries such as Turkiye. Although MAE and RMSE show Gumbel CARR and Weibull CARR have the minimum error statistics, respectively, the HMAE and HRMSE statistics indicate that among the range-based models, the EWMA model, in which the lambda parameter is estimated, performs better. Furthermore, we find that Exponential CARR according to RMSE and MAE statistics, and Weibull CARR according to HMAE and HRMSE statistics appear as the return-based volatility models with minimum error.

Keywords

CARR, EWMA, GARCH, Volatility, USD-TRY

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Introduction

The volatility studies have an extensive area in the field of financial economics. Modern Portfolio Theory (Markowitz, 1952) has provided the basis for using volatility as a risk measure. The fact that the return of an asset is assumed as a function of risk in financial studies has led to the increasing importance of modeling volatility. The changes in the volatility of a financial asset affect the asset pricing models used for obtaining equilibrium prices. Therefore, the derivative pricing methods are depending on the most accurate volatility predictions while the mean-variance analysis is a basis for investment management. Volatility is a natural consequence of the trade that takes place with the arrival of news and the subsequent reactions of investors. After reaching information to the markets, the successive movements of market actors will force the price to reach the equilibrium point. The updates of expectations and the subsequent positions of market actors will be reflected in the liquidity of a market. Since the information flow is continuous, information, liquidity, and volatility are expected to be related. This expected relationship reveals the characteristic features of financial time series. Mandelbrot (1963) states that the financial time series has no autocorrelated increments and are not usually stationary, but their squares present autocorrelation. Also, the financial time series are non-normal distributed because of the leptokurtic shape. Moreover, he shows that there exist volatility clusters which are characteristic of financial returns. The source of the volatility clusters is the direction and magnitude of the price changes. Volatility clustering occurs towards major/minor price changes after major/minor changes in both directions. Volatility is first explained by the standard deviation. However, it is not sufficient to express volatility only as a standard deviation. Because today, the assumption that the variance is constant for the variables of financial markets has lost its validity.

Thus, volatility modeling is important since it is a tool to measure risk in financial markets. It has special significance in terms of determining the risks that may arise especially in emerging financial markets such as Turkey. One of the most significant risk factors in the Turkish financial markets is the volatility that occurs in exchange rates these days. Excessive fluctuations in the exchange rate cause a delay in investment decisions and cause uncertainty in the economy. This kind of uncertainty affects investment and investor confidence, productivity, consumption, international trade, and capital flows, thus negatively affecting economic growth. Exchange rate volatility causes a high degree of uncertainty in ensuring price stability and economic growth, and in determining macroeconomic and monetary policy targets. Finding the causes of exchange rate volatility due to possible negative effects is important in terms of developing appropriate economic policies that will minimize fluctuations. Therefore, estimating exchange rate volatility is major in Turkey. The forecasting power of empirical models has a key role in hedging the risk. For these reasons, we utilize the Exponential Weighted Moving Average (EWMA) and Conditional Autoregressive

Range (CARR) models alongside the widely applied Generalized Autoregressive Conditional Heteroskedastic (GARCH)-type models in this study. The paper is organized as follows. In the second part, the volatility models are introduced briefly. Then, the study continues with the empirical results of volatility models obtained from the daily range and return series of the USD-TRY exchange rate. In the fourth section, there are model comparisons. Lastly, the fifth section concludes the study.

Volatility Models

One can model the volatility by applying various time series analysis methods. Poon and Granger (2003, 2005) divide volatility models into three groups. These groups are Stochastic Volatility models, Predictions Models Based on Past Standard Deviations, and ARCH Class Conditional Volatility models. All types of models are not considered in this study.

We mention the Historical Average and the Moving Average methods in the following since these models are the basis for Exponential Weighted Moving Average (EWMA) model which is utilized in this study. Each observation has the same weight in the Historical Average method. The method has a mean-reversion feature and assumes that volatility eventually returns to the long-run mean. The Moving Average (MA) method is made by a rolling window with a constant weighting ratio. The MA method discards the older estimates and concentrates on the last period in volatility. It loads more information on the last period. The volatility will become more sensitive to short-term fluctuations if a shorter period is chosen. EWMA model also makes predictions based on historical data. The most important feature of the method is that it gives more weight to recent observations. In other words, the model is designed with the help of an exponential weighting function so that the effect of the last day is more effective in the forecast of the next day (Poon & Granger, 2003, 2005).

Engle (1982) has opened a new page in volatility modeling in his study using UK inflation data. He introduced the Autoregressive Conditional Heteroskedastic (ARCH) model in which the conditional variance is a function of the squares of the residuals obtained from a conditional mean equation. Although the estimation of the conditional variance can model the volatility clustering that is frequently encountered in financial series, the ARCH model is insufficient to capture some of the stylized facts of financial time series. Therefore, Bollerslev (1986) developed the Generalized ARCH (GARCH) model where the conditional variance in the instant period depends not only on the historical values of the error terms but also on the conditional variances in the past. Later, many versions of GARCH-type models took their place in the literature. This study covers the most important GARCH-type volatility models.

Later, Engle and Russell (1998) introduced the Autoregressive Conditional Time (ACD) model and Chou (2005) introduced the Conditional Autoregressive Range

(CARR) model to the literature. The ACD and CARR models use the idea of GARCH models to examine the dynamic nature of the adjusted duration and range, respectively. Since duration and range are necessarily non-negative, the ACD and CARR models are also used to model time series consisting of positive observations. Therefore, the CARR model is essentially an ACD model (Tsay, 2009). We include only the CARR model in this study, as the ACD model uses the time interval between consecutive transactions in irregularly spaced intraday financial data.

The following sections consist of brief introductions to the GARCH-type, EWMA, and CARR-type models, respectively.

GARCH-type Models

The log-return of the financial time series r_t is stated as

$$r_t = \mu_t + \varepsilon_t \tag{1}$$

where μ_t is conditional mean and ε_t is residuals. The residuals can be expressed

$$\varepsilon_t = \sigma_t z_t, \quad z_t \sim f_v(0,1) \tag{2}$$

where σ_t and z_t are volatility process and innovation process respectively and $f_v(0,1)$ represent probability density function that has zero mean and unit variance. In a non-normal distribution case, v is a set of additional distributional parameters which are used for the scale and shape of the distribution.

Bollerslev (2010) has tried to provide an easy-to-use encyclopedic-type reference guide to the long list of ARCH acronyms. Although he has listed well over 100 variants of the original model, we define GARCH model extensions using Hentschel's approach in the "rugarch" package of Ghalanos (2020a; 2020b) which is an R implementation. The GARCH-family approach allows the decomposition of the residuals in the conditional variance equation to be driven by different powers for residuals and conditional variance (Hentschel, 1995). Further, it subsumes more general and widely applied GARCH-type models. Hentschel's GARCH-family equation is

$$\sigma_t^{\xi} = \omega + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^{\xi} \left[|z_{t-i} - \eta_{2i}| - \eta_{1i}(z_{t-i} - \eta_{2i}) \right]^{\delta} + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^{\xi} \tag{3}$$

with σ_t denoting the conditional standard deviation, ω the intercept, and z_t the standardized residuals from the mean filtration process. The shape of the Box-Cox transformation for the conditional standard deviation is determined by ξ, and the parameter δ transforms the absolute value function which is subject to rotations and shifts through the η_{1i} and η_{2i} parameters respectively. Applying the Hentschel approach given in Equation 2.3, the GARCH model and its extensions are obtained as follows

- GARCH if $\xi = \delta = 2$ and $\eta_{1i} = \eta_{2i} = 0$ (Bollerslev, 1986).
- Integrated GARCH (IGARCH) if $\xi = \delta = 2, \eta_{1i} = \eta_{2i} = 0,$ (Engle & Bollerslev, 1986).
- Absolute Value GARCH (AVGARCH) if $\xi = \delta = 1$ and $|\eta_{1i}| \leq 1$ (Taylor,1986; Schwert, 1990).
- Exponential GARCH (EGARCH) if $\xi = 0, \delta = 1$ and $\eta_{2i} = 0$ (Nelson, 1991).
- Nonlinear GARCH (NGARCH) if $\xi = \delta$ and $\eta_{1i} = \eta_{2i} = 0$ (Higgins & Bera, 1992).
- Nonlinear Asymmetric GARCH (NAGARCH) if $\xi = \delta = 2$ and $\eta_{1i} = 0$ (Engle & Ng,1993).
- GJR-GARCH if $\xi = \delta = 2$ and $\eta_{2i} = 0$ (Glosten et al., 1993).
- Asymmetric Power ARCH (APARCH) if $\xi = \delta, \eta_{2i} = 0$ and $|\eta_{1i}| \leq 1$ (Ding et al., 1993).
- Threshold GARCH (TGARCH) if $\xi = \delta = 1, \eta_{2i} = 0$ and $|\eta_{1i}| \leq 1$ (Zakoian, 1994).
- ALLGARCH if $\xi = \delta$ (Hentschel, 1995).

One can see the studies of Ghalanos (2020a, 2020b) and Arı (2020, 2021b) for details on GARCH-type models. The information criteria and log-likelihood values are used to evaluate the GARCH-type models (Arı, 2020; 2021b). The conditional distributions for innovations of conditional variance models are Normal (norm), Skewed-Normal (snorm), Student-t (std), Skew Student-t (sstd), Generalized Error Distribution (ged), Skewed-GED (sged), Normal Inverse Gaussian (nig) and Johnson's SU (jsu) distributions.

EWMA Model

The prominent measurement in the RiskMetrics Technical Report published by JP Morgan is the concept of value at risk (VaR). VaR is a measure of the loss that may occur in the portfolio at a given probability in a given time horizon. The VaR calculation is based on the asset volatility estimation. The RiskMetrics Technical Document (1996) uses the Exponential Weighted Moving Average (EWMA) model to estimate volatility for VaR calculation. This model has two principles asset returns are distributed symmetrically and independently, and the volatility changes depending on time. EWMA works depending on the lambda parameter, which is known as the decay factor and takes a value between 0 and 1. RiskMetrics recommends that lambda is 0.94 for daily and 0.97 for monthly observations.

We construct the EWMA model based on the study of Tsay (2012). Let $\{r_1, \ldots, r_t\}$ be a sequence of data then the EWMA of the sample with a specified lambda is

$$\hat{r}_{t+1} = \frac{r_t + \lambda r_{t-1} + \lambda^2 r_{t-2} + \cdots + \lambda^{t-1} r_1}{1 + \lambda + \lambda^2 + \cdots + \lambda^{t-1}} \tag{4}$$

where $0 < \lambda < 1$. We can infer from Equation 2.4 that the weights of the last observations are higher and weights decay exponentially in the point prediction of $\hat{r}_{t+1}$. Using the Maclaurin series expansion $1 + \lambda + \lambda^2 + \cdots + \lambda^{t-1} = (1 - \lambda^t)/(1 - \lambda)$, then the formula can be rewritten as

$$\hat{r}_{t+1} = \frac{(1 - \lambda) \sum_{i=0}^{t-1} \lambda^i r_{t-i}}{1 - \lambda^t}$$

For a large t, $\lambda^t \to 0$,

$$\hat{r}_{t+1} = (1 - \lambda) \sum_{i=0}^{t-1} \lambda^i r_{t-i} = (1 - \lambda) r_t + (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i r_{t-1-i}$$

So, we can conclude the point prediction as follows

$$\hat{r}_{t+1} = (1 - \lambda) r_t + \lambda \hat{r}_t \tag{5}$$

where $(1 - \lambda)$ and λ denote the effect of recent observation and persistence of the prediction respectively. The Equation 2.5 allows the prediction of $\hat{r}_{t+1}$ with an initial $\hat{r}_1$. Hull (2018) updates Equation 2.5 to estimate the volatility. So, the formula turns to the EWMA model for volatility that is

$$\hat{\sigma}_t^2 = (1 - \theta) \varepsilon_{t-1}^2 + \theta \sigma_{t-1}^2 \tag{6}$$

where σ_{t-1} is volatility at the end of time $t - 1$ and ε_{t-1} is the last change in the market, in other words return on the last period. Moreover, we can define the EWMA model as a special case of the IGARCH (1,1) model (Bollerslev, 2010; Tsay, 2012; Unstarched, 2014). The volatility innovations have infinite persistence is the assumption of the IGARCH model. This assumption may appear theoretically tenuous. But it may be reasonable for short-term volatility forecasting. IGARCH (1,1) is

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where $\alpha_1 + \beta_1 = 1$ and $\omega > 0$. The EWMA model is effectively a restricted IGARCH model. If the intercept ω is equal to zero and the decay parameter λ is equivalent to the autoregressive parameter β, IGARCH (1,1) reduces to EWMA.

CARR-type Models

It is well known in statistics that range is one of the good estimates of the standard deviation of a random variable. The range value of the distribution of any random variable is proportional to the standard deviation. Parkinson (1980) introduces the first range-based volatility approach. He measures daily volatility utilizing the daily range of the high/low prices and shows that the use of extreme values provides a better estimate.

Chou (2005) develops a range-based volatility model, the Conditional Autoregressive Range (CARR). This model is like standard volatility models such as the GARCH model. A distinct difference between the two models is that the GARCH model uses the rate of return as a measure of volatility, and the CARR model uses range as a measure of volatility. The CARR model is a simple and effective tool for analyzing the volatility clustering feature relative to the GARCH models. In addition, the model is successful in estimating volatility during periods of downward trends (Quiros & Izquierdo, 2011). Exponential CARR, Weibull CARR, CARR-X (Chou, 2005), Asymmetrical CARR (Chou, 2006), Gumbel CARR (Demiralay & Bayraci, 2015), Lognormal CARR (Chiang et al., 2014), Gamma CARR (Xie & Wu, 2017), and Feedback Asymmetric CARR (Xie, 2018) are countable as variants introduced into the literature. The dynamic specification of the CARR(p,q) model constructed for the range values of a time series is as follows.

$$R_t = \boldsymbol{\lambda}_t z_t, \quad z_t \sim f(1, \varsigma)$$

$$\boldsymbol{\lambda}_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{i=1}^{q} \beta_i \; \boldsymbol{\lambda}_{t-i} \tag{7}$$

where R_t is the range and is obtained by $R_t = \max(P_\tau) - \min(P_\tau)$ for $\tau \in [t-1, t]$. R_t is calculated as the difference between the highest and lowest logarithms of the prices of a financial asset observed at time τ. $\boldsymbol{\lambda}_t$ is the conditional mean of the range up to time t. It is assumed that the distribution of the innovation term z_t is distributed by a unit-mean density function $f(.)$. In addition, the coefficients in Equation 2.7 are all positive to ensure the positivity of $\boldsymbol{\lambda}_t$.

One can follow Ratnayake (2021) for details of CARR-type models and their new extensions. This paper focuses on GCARR, ECARR, and WCARR where the distribution of innovations are Gumbel, Exponential, and Weibull respectively.

Data Set

The data set covers the daily US Dollar - Turkish Lira exchange rate between the period 2019-01-01/2021-12-06. Figure 1 demonstrates the Daily Open, High, Low, and Close (OHLC) Prices of USD-TRY forex data. The descriptive statistics and the unit root test of the OHLC data are given in Table 1. OHLC series are not stationary according to unit root tests that are Phillips-Perron (PP) and Augmented Dickey-Fuller (ADF).

Figure 1. OHLC Prices of USD-TRY Between the Period 2019-01-01/2021-12-06.

Table 1

The Descriptive Statistics and Unit Root Tests for OHLC Prices of USD-TRY

Descriptive Statistics	**OPEN**	**HIGH**	**LOW**	**CLOSE**
Mean	7.036772	7.088366	6.997997	7.037783
Median	6.858605	6.8706	6.851	6.858205
Maximum	13.7	13.9178	13.648	13.7983
Minimum	5.1803	5.2188	5.16294	5.1802
Std. Dev.	1.3906	1.432857	1.367539	1.391394
Skewness	1.14528	1.289547	1.064084	1.148269
Kurtosis	5.474955	6.199368	5.025325	5.50525
Jarque-Bera	362.0107	537.5912	274.7548	367.6866
Probability	0	0	0	0
Phillips-Perron Unit Root Tests at Level				
With Constant	3.6122	3.3702	3.6067	3.6735
Probability	1	1	1	1
With Constant & Trend	4.0286	3.12	4.5642	4.3603
Probability	1	1	1	1
Augmented Dickey-Fuller Unit Root Tests at Level				
With Constant	3.063	3.0888	3.2989	3.2341
Probability	1	1	1	1
With Constant & Trend	2.5829	2.2577	8.2079	2.9796
Probability	1	1	1	1

Logarithmic range and returns data are calculated from the logarithmic prices of the exchange rate. We apply volatility models to both data sets but given the nature of the CARR models and their conditional probability distributions, we use the absolute values of the log returns to be able to apply these models and make the data positive. Time-series graphs of range and return data are available in Figure 2 and are followed by descriptive statistics in Table 2. Furthermore, Table 2 presents the Phillips-Perron and Augmented Dickey-Fuller unit root tests results which indicate that the mentioned series are stationary at level.

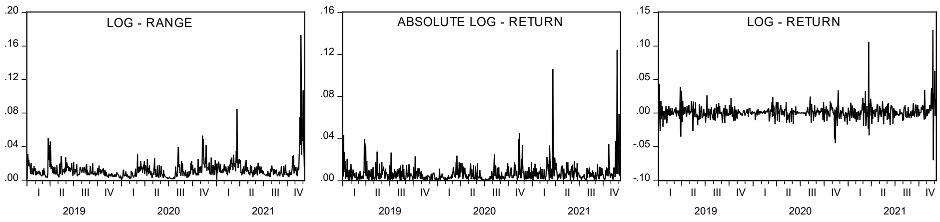

Figure 2. Daily Range, Absolute Return and Return Data of USD-TRY FOREX. Between the Period 2019-01-01/2021-12-06

Table 2

The Descriptive Statistics and Unit Root Tests for Range, Absolute Return and Return of USD-TRY FOREX

Descriptive Statistics	**Range**	**Absolute Return**	**Return**
Mean	0.011977	0.006500	0.001316
Median	0.009455	0.004076	0.000519
Maximum	0.172834	0.123864	0.123864
Minimum	0.000168	0.000000	-0.070591
Std. Dev.	0.011680	0.009069	0.011082
Skewness	6.416145	6.134065	2.806081
Kurtosis	69.23551	63.63209	36.50768
Jarque-Bera	144899.3	121818.5	36743.98
Probability	0.000000	0.000000	0.000000
Phillips-Perron Unit Root Tests at Level			
With Constant	-15.975	-21.3142	-29.3009
Probability	0.000	0.000	0.000
With Constant & Trend	-16.1799	-21.347	-29.3509
Probability	0.000	0.000	0.000
Augmented Dickey-Fuller Unit Root Tests at Level			
With Constant	-5.8528	-7.3248	-8.9284
Probability	0.000	0.000	0.000
With Constant & Trend	-6.1193	-7.5791	-9.1827
Probability	0.000	0.000	0.000

Findings

In the findings section, there are the estimation results of the GARCH-type, EWMA, and CARR-type volatility models, respectively. The evaluations of the predictive performance of the models are given at the end of this section.

The Estimation Output of GARCH-type Models

As mentioned in Section 2.1, we estimate ten GARCH-type volatility models with eight different conditional distributions. So, we compare a total of eighty models according to their log-likelihood (LLH) values alongside the Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). Since the parameters of the first thirty models fitted on the range data are statistically insignificant and the Ljung-Box (LB) and ARCH Lagrange Multiplier (LM) tests are statistically significant, the snorm-GARCH model emerges as the most appropriate model. Another reason for this evaluation is that the range data consists of positive values, and it becomes difficult to fit the conditional probability distributions used in GARCH models. The evaluation results are given in Appendix A. For the return data, the std-NGARCH of which rank is fifth is found to be the best proper model. The NGARCH models the power of conditional standard deviation which is a function of lagged conditional standard deviations and lagged absolute innovations raised to the same power. The estimation outputs of the models are given in Table 3.

One can see that the volatility persistence in both models is very high with a value of 0.99, and the half-life of the shocks is almost two years. LB and ARCH-LM tests show that there is no autocorrelation and heteroskedasticity in the standardized residuals of the models.

The Estimation Output of EWMA Models

In this study, we estimate EWMA volatility using two different approaches. In the first of these approaches, we assume the lambda value, which is the decay coefficient of the EWMA model, as fixed and estimate the volatility. The fixed lambda value is 0.94, recommended by RiskMetrics for daily data. In the other approach, the appropriate lambda value is calculated. The outputs of the estimated models applying the restricted IGARCH (1, 1) model are given in Table 4 for the range data and the return data, respectively.

As the lambda value increases, the weight of the last data decreases, and its persistence increases. The opposite is true for small lambda values. As a result, the model makes its predictions by including the coefficient of recent changes and the average weight of previous predictions. In practical studies, the interval for lambda is approximately between 0.75 and 0.98 (Tsay, 2012). However, the estimated λ=0.66 in Table 4 for the range data is outside this interval. In this case, it shows that the final data has more weight and less persistence. For the return data in Table 4, the λ=0.86 is estimated by the statistical methods, namely, the Maximum Likelihood method. Last, one can note that both model with all fixed parameters is not estimated.

LB and ARCH-LM tests for EWMA-fix for the range data show that there is autocorrelation and heteroskedasticity in the standardized residuals of the models, while these problems don’t occur in the standardized residuals of the EWMA-est and EWMA-fix models for the return data.

Table 3

The Estimation Output of GARCH-type Models

Data	Model	ω	α	β	skew	ξ	shape	AIC	BIC	LLH	LB	LM
Range	Snorm GARCH	0.00003[a]	0.64625[a]	0.35275[a]	0.66597[a]	-	-	-6.190	-6.160	2367.404	4.5570	4.1660
		(0.00001)	(0.0887)	(0.0852)	(0.0177)	-	-				[0.1923]	[0.3233]
Return	Std NGARCH	0.00007	0.3694[a]	0.7466[a]	-	1.3211[a]	3.1565[a]	-6.920	-6.890	2649.880	5.52200	1.99900
		(0.00015)	(0.0788)	(0.0384)	-	(0.3566)	(0.3738)				[0.1162]	[0.7176]

Notes: (a) denotes statistical significance at the 1% level. The values in parenthesis are standard errors. The corresponding p-values with the test statistics are in brackets. "skew" is the skewness parameter of skewed-normal distribution and "shape" is the shape parameter of the t-distribution. Please check the Equation 2.3 for the other parameters.

Table 4

The Estimation Output of EWMA Models

Data	Model	$(1-\lambda)$	λ	AIC	BIC	LLH	LB	ARCH-LM
Range	EWMA-est	0.34[a]	0.66[a]	-5.91	-5.90	2257.56	10.50	0.55
		(0.05)	(0.02)				[0.01]	[0.97]
	EWMA-fix	0.06	0.94	-5.80	-5.80	2216.69	185.90	30.42
		-	-				[0.00]	[0.00]
Return	EWMA-est	0.14[a]	0.86[a]	-6.59	-6.58	2518.02	6.81	1.82
		(0.01)	(0.01)				[0.06]	[0.76]
	EWMA-fix	0.06	0.94	-6.53	-6.53	2492.79	6.87	2.50
		-	-				[0.06]	[0.61]

Notes: (a) denotes statistical significance at the 1% level. The values in parenthesis are standard errors. The corresponding p-values with the test statistics are in brackets. The parameters of the EWMA-fix models are not estimated; therefore, the standard errors are not calculated. The values in parenthesis are standard errors. The corresponding p-values with the test statistics are in brackets.

Table 5

The Estimation Output of CARR (1,1) Models

Data	Model	ω	α	β	AIC	BIC	LLH	LB	KS
Range	GCARR	0.00057 [a] (0.00026)	0.30236 [a] (0.05377)	0.54032 [a] (0.08436)	-6.364	-6.349	-2435.090	1.37895 [0.24028]	0.06413 [0.08632]
	ECARR	0.00062 [b] (0.00034)	0.34378 [a] (0.07175)	0.54010 [a] (0.09890)	-7.088	-7.072	-2711.627	1.40194 [0.23639]	0.32591 [0.00000]
	WCARR	0.00088 [b] (0.00036)	0.42873 [a] (0.07040)	0.51837 [a] (0.08161)	-7.388	-7.373	-2826.329	1.40194 [0.23639]	0.42539 [0.00000]
Absolute Return	GCARR	0.00022 [b] (0.00008)	0.15426 [a] (0.02140)	0.76925 [b] (0.03368)	-7.358	-7.343	-2814.919	0.52323 [0.46946]	0.14660 [0.00000]
	ECARR	0.00022 [b] (0.00010)	0.17198 [a] (0.02661)	0.76487 [b] (0.03786)	-8.294	-8.278	-3172.291	0.09691 [0.75556]	0.04842 [0.33176]
	WCARR	0.00026 [b] (0.00011)	0.20373 [a] (0.03155)	0.76310 [a] (0.03836)	-8.293	-8.277	-3171.832	0.09691 [0.75556]	0.05628 [0.17769]

Notes: GCARR: Gumbel CARR, ECARR: Exponential CARR, WCARR: Weibull CARR. (a) and (b) denote statistical significance at the 1% and 5% levels, respectively. The values in parenthesis are standard errors. The corresponding p-values with the test statistics are in brackets.

The Estimation Output of CARR-type Models

We estimate range-based and absolute return-based volatilities using the Type-2 Gumbel, Exponential, and Weibull probability distributions in CARR-type models. Since the Type-2 Gumbel, Exponential, and Weibull probability distributions are defined in positive real values, absolute log-return data are used in the models. Ljung–Box (LB) test for the autocorrelation and Kolmogorov-Smirnov (KS) test for empirical distribution are applied to the residuals. All estimation outputs are presented in Table 5.

Table 5 demonstrates that all the model parameters are statistically significant at the 1% and 5% levels. The values of parameters for both data imply that the volatility shock impacts in the short term the higher for the range data. parameter shows that the long-term effects of shocks on the absolute return are highly persistent. The is less than one so that the processes are covariance stationery. Further, the LB statistics are evidence of no serial correlations in the standardized innovations of the models. According to the results of KS tests, only Gumbel is the suitable distribution for the model residuals of the range-based CARR models. In addition, the KS results substantiate that Exponential and Weibull are convenient distributions for the return-based models. In conclusion, these results indicate that the employing dynamic structures are adequate and range-based GCARR, return-based ECARR, and return-based WCARR are correctly specified.

The Comparison of Out-of-Sample Forecast Performance of The Volatility Models

The time-series graphs of the volatilities obtained from the range-based and return-based models are shown in Figure 3. When we look at the range-based models, it is seen that the models have the same pattern, except for the fixed-parameter EWMA model. This situation is the same in return-based models. All models seem to catch almost all major shocks. Naturally, the use of different data sets such as range and return, as well as the use of different probability distributions, causes the volatility sizes obtained to be different.

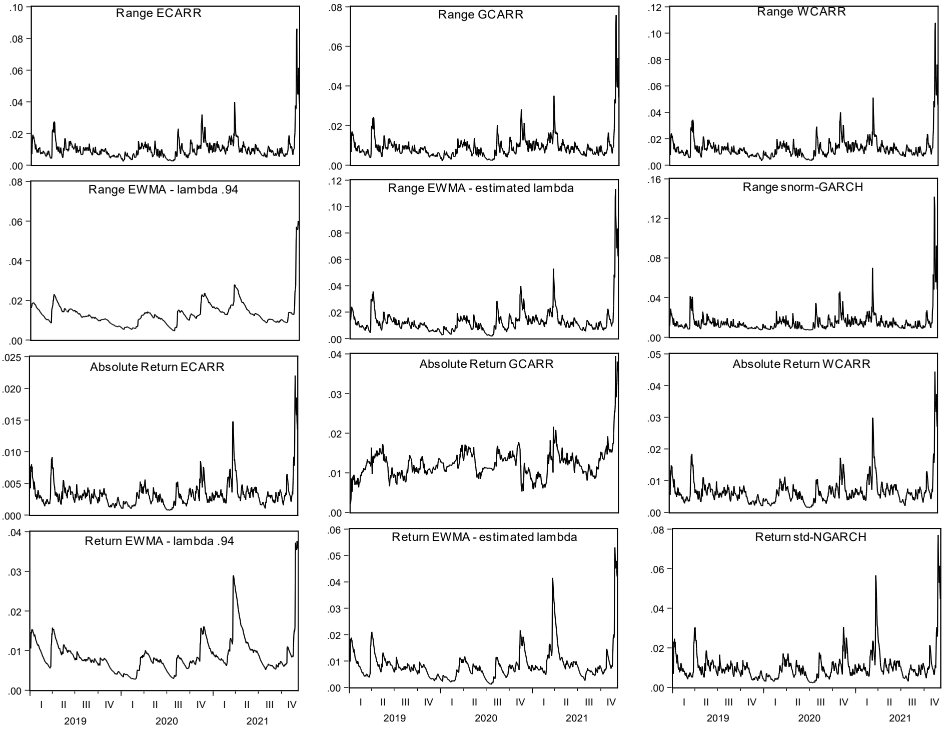

Figure 3. The Time Series Plot of the Estimated Volatilities for Range and Return Data.

We applied Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Heteroskedastic Root Mean Square Error (HRMSE), and Heteroskedastic Mean Absolute Error (HMAE) to compare the out-of-sample forecasting performance of the volatility models. Chou (2005) compares the out-of-sample forecast performance of

ECARR and GARCH models using the RMSE and MAE which are popular measures. But Bayraci and Ünal (2014) indicate that when volatility clustering occurs RMSE and MAE are not sufficient for accurate model comparison. Therefore, in addition to RMSE and MAE, we utilize HRMSE and HMAE which measure the error as an average relative error and takes high and low volatility periods into account (Bayraci and Ünal, 2014; Bollen, 2014). The error statistics can be computed using the following formulas

$$RMSE = \left(\frac{1}{n}\sum_{t=1}^{n}(\sigma_t - \hat{\sigma}_t)^2\right)^{1/2} \qquad HRMSE = \left(\frac{1}{n}\sum_{t=1}^{n}\left(\frac{\sigma_t}{\hat{\sigma}_t} - 1\right)^2\right)^{1/2}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|\sigma_t - \hat{\sigma}_t| \qquad HMAE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\sigma_t}{\hat{\sigma}_t} - 1\right|$$

where σ_t represents the range and the absolute log-returns respectively as a proxy for the true volatility, $\hat{\sigma}_t$ represents the h-step out-of-sample forecast of volatility, and n is the size of the forecasting horizon.

Table 6

Comparison of Volatility Models

Data	Model/Error	MAE	RMSE	HMAE	HRMSE
Range	GCARR	0.033162	0.053755	0.831872	1.344170
	ECARR	0.033237	0.051636	0.718184	1.128438
	WCARR	0.034223	0.050122	0.593239	0.881575
	EWMA-fix	0.035469	0.052167	0.574214	0.864303
	EWMA-est	0.040401	0.053105	0.564173	0.709144
	snorm-GARCH	0.039188	0.061293	1.072866	1.669907
Absolute Return	GCARR	0.011547	0.012623	0.816009	0.872919
	ECARR	0.006124	0.007354	1.127738	1.310347
	WCARR	0.006886	0.008424	1.046562	1.164102
Return	EWMA-fix	0.024461	0.025356	0.906028	0.919786
	EWMA-est	0.019812	0.022313	0.860581	0.894383
	std-NGARCH	0.035798	0.035966	0.941761	0.946509

Notes: The number of decimal places is given as 6 digits to present the comparison results more clearly. The minimum error statistics are colored grey.

Table 6 presents the error statistics of the models based on the range and return data. 764 observations between 2019-01-01/2021-12-06 are used for model fit. The out-of-sample forecast is applied via a 1-step-ahead forecast for the 19-day forecast horizon. Although MAE and RMSE show GCARR and WCARR have the minimum error statistics, respectively, we prefer to use the HMAE and HRMSE statistics.

So, among the range-based models, we see that the EWMA model, in which the lambda parameter is estimated, performs better according to the HMAE and HRMSE statistics. Among the return-based volatility models, ECARR according to RMSE and MAE statistics, WCARR according to HMAE and HRMSE statistics emerge as models with minimum error.

Concluding Comments

Although RMSE and MAE are widely used in error statistics comparisons, they do not contain any adjustments for heteroskedasticity, which is the general characteristic of financial data. Andersen et al. (1999) state that the heteroskedastic adjusted measures HRMSE and HMAE give different results. This statement occurs when comparing range-based volatility models in this study. Although the dataset is range-based and the MAE and RMSE statistics show that the GCARR and WCARR models have the lowest error, respectively, the HRMSE and HMAE statistics indicate that the EWMA model with an estimated parameter has superior predictive power.

The estimated $\lambda=0.66$ value of the EWMA model gives better results than the $\lambda=0.94$ value suggested by the RiskMetrics group for daily data. Of course, the fact that the data is range-based, in other words, it consists of only positive-valued observations can cause this situation. The results that the CARR-type models produce sharper volatility estimates than the GARCH-type models commonly used in the literature are also revealed in this study. The positive valued range data makes it difficult to fit the probability distributions followed by the innovations of GARCH-type models.

The absolute log return of USD-TRY is used because the probability distributions of CARR-type models are defined on positive real numbers. In return-based volatility models, CARR-type models seem to give better forecasting results. Again, the EWMA model, in which the lambda parameter is estimated and is found to be 0.86, has fewer forecasting errors than the std-NGARCH (1,1) model. It turns out that the predictive power of GARCH-type models is low, whether for range-based or return-based error measurements.

When the range data and the absolute log-return data are examined, it is seen that they have a similar structure, even though the skewness and kurtosis values are the same. It is therefore beneficial to apply CARR-type models to both datasets. Moreover, if the time-frequency of the data is changed to weekly it is expected that the forecasting performances of the CARR-type models increase more.

This study:

- provides benefits for policymakers, academics, and practitioners in emerging markets such as Turkey where exchange rate volatility is high.

- recommends comparing models using daily, weekly, and monthly data with different forecasting horizons to find the optimal lambda (decay factor) value of EWMA-filtered USD-TRY exchange rate volatility.
- particularly supports the use of CARR-type volatility models, which do not have many applications in the literature.
- encourages the use of CARR-type models based on the assumption of different probability distributions in future studies.

recommends comparing the ACD, GARCH, and Lévy Driven Continuous GARCH models using not equally spaced intraday data in future studies.

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APPENDIX A.

Table A1

The Ranking of GARCH-type Models

Range Data					Return Data				
#	**Model**	**AIC**	**BIC**	**LLH**	**#**	**Model**	**AIC**	**BIC**	**LLH**
1	nig - EGARCH11	-7.61407037	-7.577641827	2914.574881	1	std - EGARCH11	-6.935681097	-6.905323978	2654.430179
2	nig - TGARCH11	-7.564990687	-7.528562144	2895.826443	2	sstd - EGARCH11	-6.935681097	-6.905323978	2654.430179
3	nig - AVGARCH11	-7.562619498	-7.520119532	2895.920648	3	jsu - EGARCH11	-6.934237051	-6.897808508	2654.878554
4	nig - GJRGARCH11	-7.426696367	-7.390267825	2842.998012	4	nig - EGARCH11	-6.931284946	-6.894856403	2653.750849
5	jsu - EGARCH11	-7.367186182	-7.33075764	2820.265122	5	std - NGARCH11	-6.923766587	-6.893409468	2649.878836
6	jsu - TGARCH11	-7.324115513	-7.28768697	2803.812126	6	sstd - NGARCH11	-6.923766587	-6.893409468	2649.878836
7	jsu - AVGARCH11	-7.321496679	-7.278996712	2803.811731	7	std - TGARCH11	-6.922032009	-6.89167489	2649.216228
8	jsu - GJRGARCH11	-7.220987686	-7.184559143	2764.417296	8	sstd - TGARCH11	-6.922032009	-6.89167489	2649.216228
9	sged - EGARCH11	-7.009749958	-6.973321415	2683.724484	9	std - APARCH11	-6.921414544	-6.884986001	2649.980356
10	sged - GJRGARCH11	-6.993173566	-6.956745023	2677.392302	10	sstd - APARCH11	-6.921414544	-6.884986001	2649.980356
11	jsu - NGARCH11	-6.922591754	-6.886163211	na	11	jsu - TGARCH11	-6.920890245	-6.884461702	2649.780074
12	nig - IGARCH11	-6.920643462	-6.896357767	2647.685802	12	std - AVGARCH11	-6.920673415	-6.884244872	2649.697244
13	nig - GARCH11	-6.917994401	-6.887637282	2647.673861	13	sstd - AVGARCH11	-6.920673415	-6.884244872	2649.697244
14	sged - APARCH11	-6.901131314	-6.858631347	na	14	jsu - AVGARCH11	-6.920373943	-6.877873976	2650.582846
15	sged - NAGARCH11	-6.899452824	-6.863024281	na	15	jsu - APARCH11	-6.920305777	-6.877805811	2650.556807
16	nig - NGARCH11	-6.875706243	-6.8392777	2632.519785	16	std - ALLGARCH11	-6.918992729	-6.876492763	2650.055223
17	sged - ALLGARCH11	-6.86829602	-6.81972463	na	17	sstd - ALLGARCH11	-6.918992729	-6.876492763	2650.055223
18	sged - AVGARCH11	-6.827931255	-6.785431288	2615.269739	18	jsu - ALLGARCH11	-6.918287243	-6.869715852	2650.785727
19	jsu - IGARCH11	-6.7256314	-6.701345704	2573.191195	19	std - IGARCH11	-6.91775238	-6.899538108	2645.581409
20	jsu - GARCH11	-6.722831542	-6.692474423	2573.121649	20	sstd - IGARCH11	-6.91775238	-6.899538108	2645.581409
21	sged - IGARCH11	-6.698237566	-6.67395187	2562.72675	21	nig - APARCH11	-6.917283119	-6.874783152	2649.402151
22	sged - GARCH11	-6.6916184	-6.661261281	2561.198229	22	jsu - IGARCH11	-6.917268931	-6.892983236	2646.396732
23	sged - TGARCH11	-6.418484308	-6.382055765	2457.861006	23	nig - TGARCH11	-6.917137205	-6.880708662	2648.346412
24	sged - NGARCH11	-6.402518027	-6.366089484	2451.761886	24	nig - AVGARCH11	-6.917088595	-6.874588628	2649.327843
25	snorm - NAGARCH11	-6.191767782	-6.161410663	2370.255293	25	ged - EGARCH11	-6.916529378	-6.886172259	2647.114223
26	snorm - GJRGARCH11	-6.191580156	-6.161223037	2370.183619	26	nig - IGARCH11	-6.915544659	-6.891258964	2645.73806
27	snorm - NGARCH11	-6.1908399	-6.160482781	2369.900842	27	nig - ALLGARCH11	-6.915423535	-6.866852145	2649.69179

Table A1

Continue

Range Data					Return Data				
#	Model	AIC	BIC	LLH	#	Model	AIC	BIC	LLH
30	snorm - ALLGARCH11	-6.186944595	-6.144444628	2370.412835	30	jsu - GARCH11	-6.91451277	-6.884155651	2646.343878
31	snorm - GARCH11	-6.186920975	-6.162635279	2367.403812	31	sged - EGARCH11	-6.914330775	-6.877902232	2647.274356
32	snorm - EGARCH11	-6.182848739	-6.15249162	2366.848218	32	std - GJRGARCH11	-6.912799367	-6.882442248	2645.689358
33	snorm - TGARCH11	-6.17936516	-6.149008041	2365.517491	33	sstd - GJRGARCH11	-6.912799367	-6.882442248	2645.689358
34	snorm - AVGARCH11	-6.176747184	-6.140318641	2365.517424	34	nig - GARCH11	-6.912778149	-6.88242103	2645.681253
35	ged - IGARCH11	-5.979084666	-5.960870395	2287.010343	35	std - NAGARCH11	-6.912735752	-6.882378633	2645.665057
36	ged - GARCH11	-5.976457082	-5.952171386	2287.006605	36	sstd - NAGARCH11	-6.912735752	-6.882378633	2645.665057
37	std - IGARCH11	-5.975478615	-5.957264344	2285.632831	37	jsu - GJRGARCH11	-6.912413235	-6.875984692	2646.541856
38	sstd - IGARCH11	-5.975478615	-5.957264344	2285.632831	38	jsu - NAGARCH11	-6.912368651	-6.875940108	2646.524825
39	ged - NGARCH11	-5.974703958	-5.944346839	2287.336912	39	nig - NAGARCH11	-6.910679912	-6.87425137	2645.879727
40	ged - NAGARCH11	-5.974181137	-5.943824018	2287.137194	40	nig - GJRGARCH11	-6.910586732	-6.874158189	2645.844131
41	std - EGARCH11	-5.974074636	-5.943717517	2287.096511	41	ged - IGARCH11	-6.906690747	-6.888476475	2641.355865
42	sstd - EGARCH11	-5.974074636	-5.943717517	2287.096511	42	sged - IGARCH11	-6.904155429	-6.879869734	2641.387374
43	ged - GJRGARCH11	-5.973914097	-5.943556978	2287.035185	43	ged - GARCH11	-6.90393878	-6.879653085	2641.304614
44	ged - ALLGARCH11	-5.973130707	-5.93063074	2288.73593	44	ged - APARCH11	-6.903355967	-6.866927424	2643.081979
45	std - GARCH11	-5.972810311	-5.948524616	2285.613539	45	ged - NAGARCH11	-6.901929539	-6.87157242	2641.537084
46	sstd - GARCH11	-5.972810311	-5.948524616	2285.613539	46	ged - GJRGARCH11	-6.901582324	-6.871225205	2641.404448
47	std - NGARCH11	-5.972253066	-5.941895947	2286.400671	47	sged - GARCH11	-6.901407731	-6.871050612	2641.337753
48	sstd - NGARCH11	-5.972253066	-5.941895947	2286.400671	48	sged - APARCH11	-6.901131314	-6.858631347	2643.232162
49	ged - APARCH11	-5.972086159	-5.935657616	2287.336913	49	ged - AVGARCH11	-6.901015148	-6.864586605	2642.187786
50	norm - IGARCH11	-5.971834576	-5.959691729	2283.240808	50	ged - TGARCH11	-6.900677796	-6.870320677	2641.058918
51	std - GJRGARCH11	-5.97132341	-5.940966291	2286.045543	51	sged - AVGARCH11	-6.899573282	-6.857073316	2642.636994
52	sstd - GJRGARCH11	-5.97132341	-5.940966291	2286.045543	52	sged - NAGARCH11	-6.899452824	-6.863024281	2641.590979
53	std - NAGARCH11	-5.970984777	-5.940627658	2285.916185	53	sged - GJRGARCH11	-6.899096805	-6.862668262	2641.454979
54	sstd - NAGARCH11	-5.970984777	-5.940627658	2285.916185	54	sged - TGARCH11	-6.898828589	-6.862400046	2641.352521
55	ged - EGARCH11	-5.970784139	-5.94042702	2285.839541	55	ged - ALLGARCH11	-6.870501568	-6.828001601	2631.531599
56	std - TGARCH11	-5.97006291	-5.939705791	2285.564032	56	sged - ALLGARCH11	-6.86829602	-6.81972463	2631.68908

Table A1

Continue

Range Data					Return Data				
#	**Model**	**AIC**	**BIC**	**LLH**	**#**	**Model**	**AIC**	**BIC**	**LLH**
59	sstd - APARCH11	-5.969568039	-5.933139496	2286.374991	59	norm - EGARCH11	-6.702309565	-6.67802387	2564.282254
60	ged - TGARCH11	-5.969224828	-5.938867709	2285.243884	60	snorm - NAGARCH11	-6.70129316	-6.670936041	2564.893987
61	norm - GARCH11	-5.969190339	-5.950976067	2283.230709	61	snorm - GARCH11	-6.7003542	-6.676068504	2563.535304
62	norm - NGARCH11	-5.968266512	-5.943980817	2283.877808	62	snorm - GJRGARCH11	-6.69834067	-6.667983551	2563.766136
63	std - ALLGARCH11	-5.968185339	-5.925685372	2286.846799	63	snorm - NGARCH11	-6.697828299	-6.66747118	2563.57041
64	sstd - ALLGARCH11	-5.968185339	-5.925685372	2286.846799	64	norm - IGARCH11	-6.696955544	-6.684812696	2560.237018
65	std - AVGARCH11	-5.967445053	-5.93101651	2285.56401	65	snorm - APARCH11	-6.695770728	-6.659342185	2563.784418
66	sstd - AVGARCH11	-5.967445053	-5.93101651	2285.56401	66	norm - NAGARCH11	-6.694480671	-6.670194976	2561.291616
67	norm - NAGARCH11	-5.967322073	-5.943036378	2283.517032	67	norm - GARCH11	-6.694116151	-6.675901879	2560.15237
68	norm - EGARCH11	-5.967155561	-5.942869866	2283.453424	68	snorm - ALLGARCH11	-6.692072371	-6.649572404	2563.371646
69	norm - GJRGARCH11	-5.967032684	-5.942746989	2283.406485	69	norm - GJRGARCH11	-6.691764735	-6.66747904	2560.254129
70	ged - AVGARCH11	-5.966606933	-5.930178391	2285.243849	70	norm - NGARCH11	-6.691684552	-6.667398856	2560.223499
71	norm - ALLGARCH11	-5.966325995	-5.929897453	2285.13653	71	norm - APARCH11	-6.689259076	-6.658901956	2560.296967
72	norm - APARCH11	-5.965648696	-5.935291577	2283.877802	72	norm - ALLGARCH11	-6.687289862	-6.650861319	2560.544727
73	norm - TGARCH11	-5.964981395	-5.9406957	2282.622893	73	snorm - AVGARCH11	-6.672420984	-6.635992442	2554.864816
74	norm - AVGARCH11	-5.962363522	-5.932006403	2282.622865	74	norm - AVGARCH11	-6.668870394	-6.638513275	2552.50849
75	nig - APARCH11	na	na	na	75	snorm - TGARCH11	-6.667118586	-6.636761467	2551.8393
76	jsu - APARCH11	na	na	na	76	norm - TGARCH11	-6.661578819	-6.637293123	2548.723109
77	nig - ALLGARCH11	na	na	na	77	ged - NGARCH11	-5.974703958	-5.944346839	na
78	jsu - ALLGARCH11	na	na	na	78	sged - NGARCH11	na	na	na
79	nig - NAGARCH11	na	na	na	79	nig - NGARCH11	na	na	na
80	jsu - NAGARCH11	na	na	na	80	jsu - NGARCH11	na	na	na