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Van Hiele Levels of Geometric Thinking and Constructivist-Based Teaching Practices

Van Hiele Geometrik Düşünme ve Yapılandırmacı Temelli Öğretim Uygulamaları

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Abstract: This study aimed to establish the relationship between pre-service elementary mathematics teachers' (PEMTs) van Hiele geometric thinking levels and their constructivist-based teaching practices. In order to address the research questions framing this study, data related to the PEMTs' van Hiele geometry reasoning stages were gathered through the van Hiele Geometry Test (VHGT). In addition, constructivist-based teaching practice was examined by conducting the observation protocol named as Reformed Teaching Observation Protocol (RTOP) to the 108 PEMTs. Moreover, interviews were conducted to 15 Turkish PEMTs in order to obtain detailed information about the research question. The results of the data analysis represented that there was a statistically significant positive correlation between the PEMTs' constructivist-based teaching practice and their van Hiele geometry reasoning levels. As a conclusion, the PEMTs having high level of van Hiele geometry thinking were likely to enact their teaching practices more appropriately to the constructivist approach.
Keywords: van Hiele geometric thinking level, constructivist-based teaching, geometry teaching, pre-service teachers.

Öz: Bu çalışmanın amacı matematik öğretmen adaylarının van Hiele geometrik düşünme düzeyleri ve öğretim uygulamalarının ne derece yapılandırmacı yaklaşımı temel alarak gerçekleştirebildikleri arasındaki ilişkinin araştırılmasıdır. Bu çalışmayı yönlendiren araştırma problemlerini cevaplamak amacıyla, veriler matematik öğretmen adaylarının van Hiele geometrik düşünme düzeylerini belirlemek için van Hiele Geometri Testi kullanılmıştır. Ayrıca, öğretmen adaylarının yapılandırmacı yaklaşım temel alınarak gerçekleştirilen öğretim uygulaması için de Yenilenen Öğretimi Gözlem Protokolü'nün 108 matematik öğretmen adayına uygulanmasıyla toplanmıştır. Ayrıca, araştırma problemine ilişkin daha detaylı veriye ulaşmak için 15 matematik öğretmen adayıyla görüşme yapılmıştır. Araştırmanın bulguları, matematik öğretmen adaylarının yapılandırmacı yaklaşımı temel alarak gerçekleştirdikleri öğretim uygulamaları ile van Hiele geometrik düşünme düzeyleri arasında pozitif yönde anlamlı bir ilişki olduğunu göstermektedir. Sonuç olarak, van Hiele geometrik düşünme düzeyleri yüksek olan öğretmen adaylarının yapılandırmacı yaklaşımı temel alarak öğretim uygulamalarını daha etkili şekilde gerçekleştirebildikleri görülmüştür.
Anahtar Kelimeler: van Hiele geometrik düşünme, yapılandırmacı yaklaşım temelli öğretim, geometri öğretimi, öğretmen adayları.

Introduction

According to the views of Clements and Battista (1992), mathematics improves creative thinking and reasoning about the context of the problems related to real life that can be explained with the help of geometry (Atebe, 2008). Mathematics encourages creative and logical reasoning about problems in geometric world (Atebe, 2008). Also, there has been a close relationship between students' mathematical competencies and their levels of geometric understanding (van Hiele, 1986; French, 2004). Therefore, students should participate in the lessons providing them opportunities to acquire necessary skills and knowledge about geometry. In this respect, Pierre and Dina van Hiele investigated how students understand geometry and how their thinking about geometry becomes complex. This examination has resulted in a stage model describing students' geometric accomplishment levels. With this motivation, "the van Hieles developed a five-phase classification of instruction to help educators teach students to be more sophisticated thinkers

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about geometry" (McGlone, 2009, p. 32.). In this model, each van Hiele level (VHL) has its own characteristics (Mansi, 2003). With respect to the study of Crowley (1987), the levels and their characteristics are:

Level 1 (Visualization/ Recognition): At this level, the focus point is on the appearance of geometric constructs (McGlone, 2009). People recognize name (Walle, 2007) and judge figures with respect to their visual characteristics (van Hiele, 1999). They can also learn basic properties of shapes based on their visualization (Walle, 2007). Moreover, Walle (2007) explained that "the products of thought at level 0 are classes or groupings of shapes that seem to be alike" (p. 409).

Level 2 (Description/ Analysis): People can identify the specific characteristics of figures (Pandiscio & Knight, 2011) and think that all shapes form a class connected hierarchically rather than classified in an isolated way (Walle, 2007). However, these learned properties concerning shapes are perceived as isolated and unrelated since they are not logically ordered (van Hiele, 1999). Therefore, people at this level are not able to explain the connection among different properties and general classification of the figures (McGlone, 2009).

Level 3 (Informal deduction): This level can be also named as ordering, logical ordering and abstraction. At this level, people begin to establish relationships between and among properties. They can also make informal deductive arguments concerning the shapes and their properties (Walle, 2007).

Level 4 (Deduction): Battista and Clements (1992) explained that "students can reason formally by logically interpreting geometric statements such as axioms, definitions, and theorems" (p.428). They are also able to develop proofs and base them on the theories and axioms already known by them (McGlone, 2009). Moreover, Walle (2007) explained that "the product of thought at level 3 are deductive axiomatic systems for geometry" (p. 412).

Level 5 (Rigor): At this level, people are able to analyze different theorems, axioms, and postulates by identifying their differences (McGlone, 2009). They can also analyze the differences and relationships between different axiomatic systems by making comparisons and contrasts among them. People at this level are generally college mathematics studying geometry as a branch (Walle, 2007).

These stages are taken into account in the education system in order to educate students to have necessary skills in geometry. Teachers educate their students using their knowledge and skills acquired through teacher education programs with the aim of having citizens with geometric understanding and also mathematical competencies (Usiskin, 1982).

Theoretical framework

Teaching and teacher content knowledge

Types of knowledge that an effective teacher possesses are content knowledge, pedagogical content knowledge and curricular knowledge (Shulman, 1987). First, pedagogical content knowledge includes knowledge of strategies related to teaching a specific subject matter. Second, curricular knowledge includes knowledge related to the materials and media that teachers use during instruction and assessment. Third, Aubrey (1997) describes content knowledge as knowledge of the merit belonging to the content, topics, rules and operations and relationship among them. Teachers are responsible for having knowledge related to particular mathematical concepts that they transform to their students (Ball, 2000) and this knowledge can have the potential of affecting teachers' subject knowledge impacting their behaviors. Moreover, Muijs and Reynolds (2002) stated that teachers' content knowledge indirectly affects learner achievement. When many research are investigated related to effective education, the crucial role of teacher in teaching and learning is identified as a common factor. National Research Council

(2001), and Ball, Lubienski and Mewborn (2001) have agreed that teachers' knowledge, especially their subject matter knowledge is a general concern raising in many countries. In this respect, it can be claimed that what a teacher knows has great importance on what has happened in the classroom. In other words, the quality of mathematics teaching can be affected by teachers' knowledge of the content. Also, Burger and Shaughnessy (1986), and Geddes and Fortunato (1993) have added that instruction has a great importance on the students' acquisition of geometry knowledge and students' improvement of their van Hiele geometric thinking levels in order. Ball et al. (2001) have explained that the "assertion that teachers' own knowledge of mathematics is an important resource for teaching is so obvious as to be trivial" (p. 440). Du Plooy (1998) and Graham and Fennell (2001) have also agreed that teachers play an important role in the implementation of any curriculum successfully because teachers transform the required and emphasized content of the curriculum to the learners (Sandt & Neiuwoudt, 2003).

The importance of content knowledge for instruction has been explained in a way that "the common belief is that the more a teacher knows about a subject and the way students learn, the more effective that individual will be in nurturing mathematical understanding" (Swafford, et al., 1997, p. 467). In this respect, teachers' content knowledge is an important variable affecting their instructional behaviors. Usiskin (1987) has also emphasized that teachers at all levels of van Hiele teaching geometry always need to acquire necessary content knowledge concerning geometry in order to be effective instructors in geometry lessons. When the hierarchical and continuous nature of van Hiele levels is examined, it can be claimed that teachers need to progress through these levels by attaining properties concerning geometric thinking belonging to each level.

When the situations concerning geometry content knowledge of teachers and students are examined, many research have showed that students could not learn geometry as they should learn (Clements & Battissa, 1992; Crowley, 1987; Fuys 1985; Fuys, Geddes, & Tischler, 1988; Mayberry, 1983; Mitchelmore, 1997; NCTM, 1989; Senk, 1985; Ubuz & Ustün, 2003; Usiskin, 1982; van Hiele, 1986; van Hiele-Geldof, 1984). Glenda Lappan (1999), one of the previous presidents of NCTM, has also explained that "research shows that we can improve students' knowledge and ability to visualize and reason about the spatial world in which they live," in her article titled "Geometry, The Forgotten Strand". However, she has criticized whether students are attaining these knowledge and skills, and emphasized that the data collected by the Third International Mathematics and Science Study (TIMSS) and the National Assessment of Education Progress (NAEP) show that student's performance and achievement in the field of geometry is critical and not at the required level. The reasons of this problem have been investigated by many researchers, Usiskin (1987) and Clements (2003) have claimed that this problem results from teachers' content knowledge of geometry. They also claimed that the students' achievement in a lesson or understanding of a concept is connected to their teachers' understanding. According to Stipek (1998), although teachers' content knowledge is important in students' performance, pre-service and in-service teachers' geometry knowledge is not at the required and expected level. They could not acquire required knowledge and skills about geometric reasoning and geometric construction (Köse, Tanışlı, Erdoğan, & Ada, 2012; Napitupulu, 2001). Moreover, Mayberry (1983) claimed that pre-service elementary teachers show geometric reasoning unsuccessfully and Hershkowitz and Vinner also (1984) added that in-service teachers and their students are more likely show similar patterns of misconceptions (Pandiscio & Knight, 2011). According to the findings of these research, there are teachers who graduated from teacher education programs without having sufficient geometry content knowledge. Through the instructions, the teachers can help students learn geometry effectively considering van Hiele theory (Battista, 2007; van Hiele, 1959). In this respect, teachers taking the role of filter between curriculum and learners (Du Plooy, 1998; Graham & Fennell, 2001) should be equipped with sound knowledge and skills of geometry in order to perform instructions achieving the properties of constructivist-based teaching. In other words, the teachers possessing necessary geometry knowledge and geometric thinking skills can perform their instruction by promoting student performance.

Constructivism and importance of teacher's geometry content knowledge

In the constructivism, the recent accepted philosophy of the curriculum in Turkey, "knowledge is not transmitted directly from one knower to another, but is actively built up by the learner" (Sawada, Piburn, Turley, Falconer, Benford, Bloom, & Judson, 2000, p. 3). Furthermore, learners can form and structure their understandings. However, they do not passively absorb the understandings of others. In the consideration of these main properties of the constructivist approach, teachers are responsible for helping students to attain content knowledge without memorization and construct their own content knowledge. Also, a teacher is viewed as a person possessing specific and sufficient content knowledge (Sandt, 2007) in the constructivist approach. Moreover, teachers who do not have sufficient geometry content knowledge are not expected to enact their teaching appropriately to constructivist approach effectively. To line with this view, it is necessary to determine pre-service teachers' deficiencies in geometry content knowledge and quality of constructivist-based/reform-based teaching practice, so that they can remove these deficiencies with the help of the courses and academicians in teacher education programs. Furthermore, the relationship between the PEMTs' van Hiele geometric thinking levels and their constructivist-based teaching practice levels should be determined so as to provide beneficial suggestions for the improvement of their geometric thinking and teaching practice. Then, they will become successful instructors in geometry lessons in the future. In addition, the studies examining the connection between individuals' content knowledge and their instruction can provide contribution to the literature. Furthermore, it is beneficial to examine the PEMTs' geometry content knowledge and constructivist-based teaching practice. The results of the present study can provide contribution to the literature by determining the current levels of the PEMTs' geometry content knowledge and their relation to instruction in their teaching practices performed based on constructivism.

The rationale and the purpose of the study

Instructional practice is affected by teachers' and pre-service teachers' content knowledge because the questions of "What is taught?" and "How it is taught?" can be answered by considering teachers' knowledge and characteristics (Swafford, Jones, & Thornton, 1997). Furthermore, Nason (1996) state "subject matter knowledge would extremely limit their ability to help their learners develop integrated and meaningful understandings of mathematical concepts and processes" (p. 263). However, further research exploring the issue of required knowledge for teaching mathematics and geometry should be made (Bowning et al., 2014). Kirby (2005) emphasizes this issue by stating "the nature of the knowledge required for successful teaching of mathematics is poorly specified, and the evidence concerning the mathematical knowledge that is needed to improve instructional quality is surprisingly sparse" (p. 2). Moreover, in the literature, there have been research explaining the needed and desired content knowledge for teaching (Hill, Rowan & Ball, 2005; Ma, 1999; National Research Council, 2001). This study aimed to make contribution to this issue by focusing on geometry teaching.

According to the findings of many research, many students are not able to learn geometry and have geometry knowledge as much as they need (Clements & Battissa, 1992; Ubuz & Ustün, 2003; Usiskin, 1982; van Hiele, 1986; van Hiele-Geldof, 1984) so that many elementary school students cannot get the necessary skills and knowledge. With the aim of meeting the students' needs for the branch of geometry, teachers especially elementary teachers are expected to design appropriate learning environments so that they can help their students become successful in geometry (Gül-Toker, 2008). In this respect, it is beneficial to educate teachers to have required knowledge about geometry content for teaching it. In other words, teachers equipped with sufficient geometry knowledge and skill of designing and implementing the geometry lessons can help students get geometry knowledge by forming appropriate classroom culture. The previous research have stated the reasons of students' poor performance in geometry by focusing on weak treatment performed based on mathematics curricula, obstacles met through geometry lessons and insufficient teacher knowledge (Clements, 1999; Lehrer, 2003, Steele, 2013; Strom et al., 2001).

In this respect, it can be explained that teachers' geometry knowledge and understanding can affect their performance on designing lessons and implementing lessons, managing classroom culture and filtering curriculum to learners.

It has been claimed that pre-service elementary teachers have tended to get and represent weak geometric understanding (Mayberry, 1983, Pandiscio & Knight, 2011). Pre-service teachers are expected to graduate from their education programs with necessary geometry content knowledge to teach this knowledge. With this respect, it is important to pay attention to knowledge of geometry content and teaching them. Therefore, the pre-service teachers taking the geometry courses in their undergraduate programs participated to the study so that it might be possible to assess the efficiency of these geometry courses and their actual success in their instructions as mathematics teachers in the future. With this motivation, these pre-service teachers were ideal candidates to determine the pre-service teachers' levels of van Hiele geometric thinking and constructivist-based teaching practice.

The present study paid attention to exploring the pre-service elementary mathematics teachers' levels of Van Hiele geometric thinking, pre-service teachers' geometry teaching practices, and the relationship between pre-service elementary mathematics teachers' van Hiele geometric thinking levels and constructivist-based teaching practices with respect to the dimensions of the RTOP (lesson design and implementation, content and classroom culture). Hence, it aimed to answer the following research questions:

1. What are the pre-service elementary mathematics teachers' (PEMTs) van Hiele geometric thinking levels?
2. What is the relationship between the PEMTs' van Hiele geometric thinking levels and constructivist-based teaching practice?
    a. What is the relationship between the PEMTs' van Hiele geometric thinking levels and constructivist-based teaching practice in terms of the dimension of lesson design and implementation?
    b. What is the relationship between the PEMTs' van Hiele geometric thinking levels and constructivist-based teaching practice in terms of the dimension of content?
    c. What is the relationship between the PEMTs' van Hiele geometric thinking levels and constructivist-based teaching practice in terms of the dimension of classroom culture?

Method

Research design

Qualitative and quantitative methods were used to explore pre-service elementary mathematics teachers' van Hiele geometric thinking levels and their constructivist teaching practices. Sequential explanatory mixed method research design (Sullivan, 2009) was used in the current study in order to clarify and detail quantitative data. Through this mixed method, it became possible to focus on what the PEMTs think about the relationship between the PEMTs' geometric reasoning and their constructivist-based teaching practice. In other words, this method aimed to represent the relationship between the PEMTs' geometric thinking and constructivist teaching practices by providing detailed information based on their opinions about this relationship.

Quantitative procedures

Participants

Totally, 108 pre-service elementary mathematics teachers who were junior and senior students participated in the current study. They were enrolled in an undergraduate program of Elementary Mathematics Education at a public university in Turkey. Of these pre-service teachers, 53.7% were girls and 46.3% were boys. Also, 55.6% were junior and 44.4% were senior of these PEMTs.

These pre-service elementary mathematics teachers will become mathematics teachers in the fifth, sixth, seventh and eighth grade level of students. In addition, all students were volunteer to take role, do the test, design and implement geometry lessons in the present study.

Instruments

Two quantitative instruments; the Reformed Teaching Observation Protocol (RTOP) and van Hiele Geometry Test (VHGT) were conducted to the participants. Firstly, the RTOP was used in order to examine the pre-service teachers' constructivist-based teaching practice in geometry lessons. Necessary permissions were taken from the academicians studying the adaptation and translation of this instrument. In this study, the RTOP as a standardized instrument to measure the degree to which classroom instruction and organized with respect to constructivism (MacIssac & Falconer, 2002), the accepted educational philosophy in Turkey was used. In this respect, the RTOP was used for guiding the classroom observations as a reliable tool to acquire knowledge about geometry teaching practices enacted by the PEMTs. There exist three main factors as lesson design and implementation, content and classroom culture. The first factor of lesson design and implementation has 5 items. The second main factor of content has two subcategories as propositional knowledge including 5 items and procedural knowledge including 5 items. The third main factor of classroom culture has two sub-factors as communicative interactions with 5 items and relationship with students with 5 items. Therefore, the instrument has 25 items (Temiz & Topcu, 2011). The items are rated on a 5-point Likert scale. In this respect, a score of "1" was assigned when the particular behavior did not occur at all. A score of "5" was assigned when the particular behavior was very descriptive about the behaviors of the individual being observed. Scores ranged 0-125 points and higher points can be implemented that more constructivist-based teaching practices were observed (Sawada et al., 2000). Furthermore, this scale was translated into Turkish in a study belonging to Temiz and Topcu (2011). Temiz and Topcu (2011) confirmed that this scale is reliable and valid. Three dimensions were determined for the scale and the Cronbach Alpha reliability measures were found as .90, .86, and .91, for the first, second, and third level, respectively. In the current study, the Cronbach Alpha reliability measures were calculated as .87, .82 and .89.

The second quantitative instrument is van Hiele Geometry Test (VHGT). The Cognitive Development and Achievement in Secondary School Geometry project was (CDASSG), developed and conducted by Zalman Usiskin and Sharon Senk. In this project, Usiskin (1982) and Senk (1989) established that the van Hiele theory is beneficial to make predictions about the people's achievement and performance in geometry courses. The van Hiele theory as a commonly used theory was beneficial in examining the content knowledge of in-service and pre-service teachers. Also,van Hiele (1959) explained that this theory defines a model in order to measure people's level of geometric argumentation or thinking (Sandt & Nieuwoudt, 2003). With this motivation, VHGT, developed by Usiskin (1982) was used in order to estimate the PEMTs' content knowledge of geometry, van Hiele geometric thinking level. Furthermore, the van Hiele Geometry Test (VHGT) includes 25 multiple-choice geometry questions and is applied to the participants in 35 minutes. In this test, the first five items belonged to level 1, the second five items belonged to level 2, the third five items belonged to level 3, the fourth five items belonged to level 4, and the last five items belonged to level 5. In addition, the reliability and validity of this test was studied in many studies belonged to Burger and Shaughnessy (1986), Fuys, Geddes and Tischler (1988), Moody (1996), Moran (1993) and Usiskin (1982). Moreover, this test was translated into Turkish in a master thesis study, and sufficient values for the Cronbach Alpha reliability measures were calculated (Duatepe, 2000).

These instruments used in the present study was used in order to detail information being appropriate for the rationale of the study. Firstly, the RTOP was practically useful in rating the PEMTs' constructivist-based teaching practices. Moreover, this instrument could provide the opportunity observing general classroom actions such as facing classroom management problems, lesson closure. Moreover, this protocol can be used with the help of its training and reference manuals more effectively and clearly (MacIsaac & Falconer, 2001). In addition, each item in the observation protocol helps observers to understand what constructivist approach looks like in classroom teaching practices and the main characteristics of this approach (Henry, Murray, & Phillips, 2007). Secondly, the VHGT can provide detailed information for the study since it assesses the participants' geometry content knowledge by classifying geometric thinking into categories successfully including main topics of geometry.

Data collection

The PEMTs voluntarily participated in the current study. Then, they designed their geometry micro-teachings consistent with constructivism and implemented their micro-teachings in the courses named "Teaching Geometry" for senior students and "Mathematics Teaching Methods" for junior students. The researchers of the current study observed these micro-teachings by using the adapted form of the RTOP and guidance manual into Turkish and, each observation took 35 minutes. Data collection period concerning observations took approximately twenty weeks. Furthermore, the PEMTs were administered van Hiele Geometry Test (VHGT) consisting of 25 multiple-choice questions to measure the PEMTs' van Hiele geometric thinking levels after their micro-teachings had been completed. The data collection period about administering the test lasted approximately 35 minutes. Then, the investigators of the study read and scored all participants' answer sheets obtained from VHGT.

All participants were given a score referring to a van Hiele level from the VHGT by using Usiskin's grading system. The participants were assigned a weighted sum score by using the following manner:

- 1 point for meeting criterion on items 1-5 (level-I)
- 2 points for meeting criterion on items 6-10 (level-II)
- 4 points for meeting criterion on items 11-15 (level-III)
- 8 points for meeting criterion on items 16-20 (level-IV)
- 16 points for meeting criterion on items 21-25 (level-V)" (1982, p. 22).

For the purpose of removing the limitations of the quantitative part of the present study, some precautions were taken. Firstly, the descriptors of extreme ratings of the items in the RTOP (1 for 'Never Occurred' and 5 for 'Very Descriptive') do not indicate exactly opposite cases. Therefore, the individuals being observed might be rated wrongly. In order to remove these potential obstacles, Training Guide of the RTOP and expressions belonging to each item on the RTOP were examined. Afterward, practical activities including scoring the subjects being observed had been done carefully before the actual observations were carried out. By making this kind of practice, the researchers improved required skills to use this observation protocol. Furthermore, they became familiar with the items of the instrument. The scoring of the participants was done by more than one academician studying science or mathematics education. In order to score the PEMTs' content knowledge of geometry truly, VHGT and the related studies (Senk, 1989; Spear, 1993) especially the study of Usiskin (1982) were analyzed carefully.

Data analysis

In order to assess the PEMTs' van Hiele levels, the criterion for acquiring the properties of any given level was four out of five correct responses. In order to determine the participants' actual van Hiele levels, 100 - point numerical scale developed by Gutierrez, Jaime, and Fortuny (1991) was also used. This scale claims that there are five qualitative scales between two van Hiele levels. Moreover, Gutierrez, Jaime, and Fortuny (1991) said that "'Values in interval' (0%, 15%) means 'No Acquisition' of the level. 'Values in the interval' (15%, 40%) means 'Low Acquisition' of the level. 'Values in the interval' (40%, 60%) means 'Intermediate Acquisition' of the level. 'Values in the interval' (60%, 85%) means 'High Acquisition' of the level. Finally, "values in the interval" (85%, 100%) means "complete acquisition of the level" in the scale (p. 43). In addition, descriptive statistics analysis was used in order to determine their levels of geometry content thinking and constructivist-based teaching practice. Furthermore, Pearson multiplication of moments correlation quotient analysis was used to determine the relationship between the participants' van Hiele levels and each factor of level of constructivist-based teaching practice. The scores obtained from the RTOP were accepted as continuous ranging between 0 and 125 based on previous research (Temiz & Topcu, 2011; 2013). Also, the scores acquired through van Hiele Geometry Test was accepted as continuous based on scoring suggested by Gutierrez, Jaime, and Fortuny (1991). Hence, Pearson multiplication of moments correlation quotient analysis was used again to estimate the relationship between total score of RTOP and van Hiele levels by meeting the assumption of this test.

Qualitative procedures

Data collection and participants

The PEMTs' constructivist-based teaching practices were assessed considering the factors of the RTOP. They were observed by using the RTOP. Then, structured interviews were conducted to 15 PEMTs with the aim of acquiring more appropriate information about their opinions related to the connection of van Hiele geometric thinking levels to constructivist-based teaching practice. The interview questions were formulated considering the three main factors of the RTOP (lesson design and implementation, content, and classroom culture) and geometric thinking in order to obtain information about the PEMTs' views on them. 15 PEMTs were randomly selected from subjects being observed to be interviewed. In addition, they were asked whether they volunteered to be interviewed after being selected by the researchers randomly and they accepted to be interviewed. Each PEMT was assigned a pseudonym. Before starting to administer the interviews, the PEMTs were informed about their rights and assured confidentiality of the data. In this respect, after completing general conversation, interviews were conducted to 15 PEMTs by asking the same questions in the same order. The PEMTs were also encouraged to explain their thoughts related to the questions in detail, and to reflect and consider their answers. Each interview lasted approximately 35 minutes and was audio-recorded.

With the aim of providing evidence for the trustworthiness of the qualitative part of the present study, some precautions were taken. Validity and reliability of this part was limited to the participants' faithfully answers to the interview questions. Therefore, volunteer participants were interviewed and the purpose of the current study was explained to the interviewees in detail. Moreover, the implications made from the interviews were discussed with the interviewees by performing member checking strategy. After content analysis was conducted to the transcripts of the interviews, the researcher and the PEMTs who had been interviewed met again and discussed the appropriateness of the implications made by the researchers based on their explanations through the interviews.

Data analysis

Qualitative data were collected through interviews in the present study. The interview transcripts were analyzed with respect to the qualitative analysis procedures. Marshall and Rossman (1999) explained that there exist six steps recommended in analyzing qualitative data. These steps can be summarized as organizing the data, forming codes and themes, coding the data, testing the emergent interpretations as based on individual differences, examining for alternative expressions and preparing the report. In this study, all these steps were investigated and all interviews were analyzed according to the process including coding, ordering and displaying and conclusion drawing. In the category of coding interview transcripts were coded by focusing on research questions. Also, in the other category of ordering and displaying includes gathering information and determining patterns and themes. In the last category of conclusion drawing includes drawing conclusions with respect to collected data.

The PEMTs' answers to the interview questions lasting approximately 35 minutes were audio-taped and transcribed verbatim. Initially, the data in written format were read by the researchers. Main ideas and expressions related to the answers of each question and the participants were determined by the researchers independently. Each transcript was also analyzed independently by considering potential categories. After completing the coding process, all transcripts were coded in an iterative process. Then, the researchers made discussions by analyzing the differences and similarities about the lists of categories formed by the researchers independently and the disagreements about the categories were negotiated. After discussions were completed, they formed the list of common categories. The rate of consistency between the two researchers in deciding the codes and themes for the list was accepted as 85%. After the analysis had been completed, an academician having the Ph.D. degree in mathematics education and not taking role in the current research reviewed and evaluated the analysis with respect to consistency and coherence. According to Lincoln and Guba (1985), this qualitative data analysis process including investigator triangulation member checking provides the validity of the analysis (Topcu, 2011). Content analysis was conducted by two researchers.

Results

Pre-service elementary mathematics teachers' levels of van Hiele geometric thinking and constructivist-based teaching practice

Table 1 displays the descriptive values belonging to the participants and obtained by the VHGT and the RTOP. Also, it presents the values for each query of the RTOP scale assessing constructivist-based teaching practice of the PEMT, and the RTOP has been classified on the basis of the "lesson design and implementation", "content", and "classroom culture" subcategories. First, the value of mean was calculated as 3.13 and standard deviation was done as 0.91 in order to clarify the PEMTs' van Hiele geometric thinking levels. There are participants who attained level-I, level-II, mostly level-III and level-IV but there is not participant attained level-V. Moreover, according to the 100 - point numerical scale developed by Gutierrez, Jaime, and Fortuny (1991), the score 0.13 can take place in the interval named "No Acquisition" of the upper level. In other words, PEMT completed the level-III (Informal Deduction), but they could not reach the level-IV (Deduction). In this respect, the idea that levels of the PEMTs' van Hiele geometric thinking levels were sufficient can be claimed. Second, the value of mean was calculated as 4.29 and standard deviation was done as 0.79 in order to make clear the PEMTs' constructivist-based teaching practice levels. Hence, the PEMTs' implementation of micro-teachings was determined as a significantly high constructivist teaching practice level. In addition, in order to explain the constructivist-based teaching practice levels more clearly, the PEMTs' practice levels were indicated with a mean of 4.22 and standard deviation of 0.79 for lesson design and implementation subcategory, with a mean of 4.29 and standard deviation of 0.67 for content subcategory and with a mean of 4.32 and standard deviation of 0.76 for classroom culture subcategory based on RTOP scores.

Table 1

Descriptive Statistics for the PEMTs' Scores of VHGT and RTOP

Tests	Mean	SD
VHGT	3.13	0.91
RTOP	4.29	0.71
Subsets of the RTOP		
Lesson design and implementation	4.22	0.79
Content	4.29	0.67
Classroom culture	4.32	0.76

Relationship between pre-service elementary mathematics teachers' constructivist-based teaching practice levels and van Hiele geometric thinking levels

Table 1 displays the correlation coefficients among the van Hiele geometric thinking and the determinants of implementing constructivist-based instruction based on the main factors of RTOP. Furthermore, the relationship between van Hiele level and the total RTOP score was revealed. In other words, the "Pearson multiplication of moments correlation quotients" was calculated among the values received by the PEMTs and obtained from the VHGT and the scores accumulated from the RTOP subscales. Also, when the correlation quotients obtained for the van Hiele geometric thinking level and constructivist-based teaching practices of the PEMTs, in Table 2, are examined, it can be seen easily that there are meaningful associations between the VHGT and the RTOP (with its factors). Furthermore, it is observed that there are significantly high values as correlation quotients changing between 0.650 and 0.671 revealing the following pattern as in Table 2.

The PEMTs' constructivist-based teaching practices based on the factor of lesson design and implementation was positively correlated with van Hiele geometric thinking level ($r = 0.650$, $p < .01$) according to the scores obtained from the RTOP and the VHGT. The interview findings for the question: 'Do you think a teacher's geometry content knowledge is connected to the design and implementation of his/her lesson?' supported the quantitative results and corroborated its details. This question was used to investigate whether one would believe there was a relationship between teachers' geometry content knowledge and designing and implementing their lessons. All students agreed that geometry content knowledge was related to the quality of designing and implementing an effective lesson consistent with constructivist approach. For example:

> *I think there is a positive relationship between teachers' content knowledge of geometry and the quality of their designing and implementing their lessons. Teachers having sufficient geometry content knowledge tend to design and implement their lessons by using student-centered activities. Also, they design and implement their lessons with respect to students' ideas so that their lessons encourage students participate the lessons actively and explore the main points of the subjects. Moreover, these teachers are confident about their content knowledge of geometry and their potentials to instruct effectively.*

The PEMTs' constructivist-based teaching practices concerning content was positively correlated with van Hiele geometric thinking level ($r = 0.659$, $p < .01$). The interview findings for the question: "Do you think a teacher's geometry content knowledge was connected to their effectiveness of teaching this knowledge in his/her lesson?" encourage quantitative results and corroborate its details. This question was used to investigate whether one would believe there was a relationship between teachers' geometry content knowledge and success of teaching this knowledge in their lessons. All students affirmed that geometry content knowledge was related to the quality of teaching the knowledge of geometry content consistent with constructivist approach. For example:

> *I think that teachers' content knowledge is related to the quality of teaching this knowledge positively. Teachers having sufficient geometry content knowledge can instruct by using appropriate models, representations and materials effectively. Also, these teachers' students learn the subject without misconceptions since they provide their students sufficient and appropriate knowledge about the subject. In addition, they are more likely to use real-life examples and make connection with other disciplines consistent with the subject.*

The PEMTs' constructivist-based teaching practices concerning classroom culture was positively correlated with van Hiele geometric thinking level ($r = 0.671$, $p < .01$). The interview findings for the question: 'Do you think a teacher's geometry content knowledge is connected to the quality of interactions consistent with constructivism in his/her lesson?' encouraged the quantitative results and corroborated its details. This question was used to investigate whether one would believe there was a relationship between teachers' geometry content knowledge and the quality interactions happening at the classroom atmosphere that they created in the lessons. All students affirmed that geometry content knowledge was related to the quality of interactions at the classroom atmosphere consistent with constructivist approach. For example:

> *I think that teachers' geometry content knowledge and the quality of interactions happened in the class are positively related. Teachers having sufficient content knowledge of geometry tend to use student-centered activities such as discussion and group works and implement their lessons effectively. Therefore, students share and discuss their ideas in respect. In addition, these teachers encourage students to ask questions and discuss the subject with them. They guide and encourage their students investigate and construct their knowledge by providing appropriate atmosphere and interactions.*

In general, Pearson multiplication of moments correlation quotients value calculated significantly high (r = 0.671, $p < .01$) so it could be claimed that there was statistically significant positive correlation between the PEMTs' constructivist-based teaching practice levels and their van Hiele geometry understanding level.

Table 2

Correlations between the Pre-service Elementary Mathematics Teachers' Levels of van Hiele Geometric Thinking and Constructivist-Based Teaching Practice

	Lesson Design and Implementation	Content	Classroom Culture	RTOP
Van Hiele Geometric Thinking Level	0.650**	0.659**	0.671**	0.671**

**p< .001

Discussion, Conclusion and Implications

In the present study, it was aimed to identify the levels of the PEMTs' geometric thinking and constructivist-based teaching practice and to establish the relationships among the PEMTs' geometric thinking levels and constructivist-based teaching practices. These purposes were explored in the consideration of sub-dimensions and total dimensions of the RTOP, for constructivist-based teaching practice. The VHGT revealed satisfactory results in identifying the the PEMTs' van Hiele geometric thinking levels in the current study, similar to the expected van Hiele geometric thinking levels claimed by Hoffer (1988), Spear (1993) in a US context and YOK (2007) in a Turkish context. Congruent to the expected van Hiele geometric thinking level stated by Hoffer (1988), Spear (1993) and YOK (2007), the PEMTs achieved to reach the expected level (level-III) in the present study. This finding was confirmed Spear's (1993) claim. In the consideration of the amount of knowledge that an effective elementary mathematics teacher had, Spear (1993) stated that "all elementary school mathematics teachers and prospective elementary school mathematics teachers should at least attain the first three van Hiele levels" in order to teach effectively and enhance their students' learning beneficially in geometry lessons. In this respect, it can be explained that the PEMTs have adequate geometry knowledge to instruct effectively in the future. It was also important that they should improve their geometry knowledge by passing through level-IV. In the consideration of the PEMTs' van Hiele geometric thinking stages found in the present study, it can be concluded that the teacher education program beneficially prepares the PEMTs for teaching geometry effectively in the future.

In the consideration of assessing the PEMTs' constructivist-based teaching practices, the micro-teachings of the PEMTs were observed with the help of the RTOP. The RTOP revealed satisfactory results in identifying the PEMTs' constructivist-based teaching practices, parallel to the finding of the previous study conducted by Temiz and Topcu. Similar to Temiz and Topcu's (2011) findings in which pre-service teachers' constructivist-based teaching practice levels were high with a mean value of 3.91, the present study claimed that the PEMTs' constructivist-based teaching practice levels were high with a mean value 4.29. In this respect, it can be explained that the PEMTs have adequate knowledge and skills about teaching geometry to enact their teaching practices effectively appropriate to constructivist approach in the future. It is also important that they should improve these knowledge and skills with respect to constructivism. In the consideration of the PEMTs' constructivist-based teaching practice levels found in the present study, it can be stated that teacher education program beneficially prepares the PEMTs for teaching geometry consistent with constructivist approach effectively in the future. In other words, current teacher education programs providing the PEMTs necessary knowledge and skills about constructivist-based teaching would possibly have been beneficial for the PEMTs in the term of implementing constructivist-based instructions. This claim is congruent to the finding of the study conducted by Temiz and Topcu (2011).

In the present study, it mainly aimed to investigate the connection of the PEMTs' van Hiele geometric reasoning levels to their constructivist-based teaching practices. This connection was reported and indicated by satisfactorily high values. This study revealed that the PEMTs' van Hiele geometric thinking levels were connected to their level of constructivist-based teaching practices. By considering the number and magnitude of correlations established between van Hiele geometric understanding level and the sub-dimensions and total dimension of constructivist-based teaching practices in this study. It can be proposed that both of the variables examined in the study were interrelated. The other significant result was that interview findings for the relationships between teachers' content knowledge of geometry, van Hiele geometry understanding and the dimensions of constructivist-based teaching practice were consistent with the quantitative results of the present study. In other words, the qualitative results confirmed the quantitative results in this study. In light of these quantitative and qualitative findings, it can be stated that the PEMTs having a high level of van Hiele geometry understanding level tend to enact their teaching practice appropriate to constructivist approach effectively. This result confirms the claims of National Research Council (2001), Ball, Lubienski and Mewborn (2001) and Muijs and Reynolds (2002). In other words, teachers (Sandt & Nieuwoudt, 2003) and teachers' knowledge, particularly their subject content knowledge (Ball et.al., 2001) plays the crucial role in teaching and learning. In addition, it can be claimed that the quality of instruction is directly related to teachers' content knowledge (National Research Council, 2001) and teachers' behavior is influenced by their knowledge (Kennedy, 1998; Muijs & Reynolds, 2002). These claims can be emphasized by stating that content knowledge of teachers is a determinant of effective teaching (Kanes & Nisbet, 1996; Ferguson, 1991), teachers' method of teaching (Carpenter, Fennema, Peterson & Carey, 1988; Leinhardt & Smith, 1985) and teachers' use of the pedagogical tools (Carpenter et. al., 1988). Furthermore, it is clear that teachers are expected to be experts in their classrooms (Reinke, 1997) and a depth of content knowledge is prerequisite for effective teaching in order to provide this expectation (von Minden, Wallis & Nardi, 1998). Therefore, teachers can support adequate explanations of concepts they do not understand (National Research Council, 2001). Finally, these judgments can be summarized that the quality of mathematics and also geometry teaching depends on teachers' content knowledge (Ball, Hill & Bass, 2005). In light of the judgments, it can be suggested that when PEMTs are at Level-III or above van Hiele geometry reasoning stage, they can instruct effectively consistent with constructivist approach. Moreover, it is needed that they should improve their van Hiele geometric thinking levels and constructivist-based teaching practice by teacher education programs in the pre-service years. Therefore, they can remove their deficiencies about the content knowledge of geometry and the problems about their constructivist-based teaching practice and they improve themselves in the consideration of these both variables. To conclude, they would become an effective teacher having necessary geometry content knowledge and high and developing constructivist-based teaching practice in the future. The present study can make a contribution to establish the current status of the PEMTs' van Hiele geometric thinking levels and to provide information about their teaching quality before they are going to become real teachers in the future.

The purpose of the present study was to establish the levels of the PEMTs' van Hiele geometry understanding and their constructivist-based teaching practice. Understanding the PEMTs' quality of geometry instructions and the level of content knowledge of geometry in their pre-service years provide opportunities to remove their deficiencies and improve their knowledge of geometry and skills about teaching geometry. In this way, it can become possible to educate PEMT instructing effectively with sufficient geometry content knowledge as real teachers in the future for teacher education programs. With this motivation, the PEMTs' scores of the VHGT and the RTOP were assessed in order to determine their levels of content knowledge of geometry and constructivist-based teaching practices respectively. The participants of the present study were at Level – III of van Hiele geometry understanding and this level or above is the expected level of the PEMTs' van Hiele geometry understanding level. In addition, the PEMTs attained very high scores from the RTOP instrument so that it can be claimed that the PEMTs can instruct effectively consistent with constructivist approach.

A growing body of the present study provides evidence that there is a relationship between the PEMTs' van Hiele geometry understanding level and their constructivist-based teaching practices in the positive manner. This relationship was assessed for the PEMTs' constructivist-based teaching practice with respect to its dimensions and total dimension. The statistically significant correlation quotient values were found among the dimensions: lesson design and implementation, content and classroom culture and total dimension of constructivist-based teaching practice and their van Hiele geometry understanding. Further, qualitative findings obtained from the interviews are consistent with these quantitative findings. In this respect, it can be suggested that the PEMTs having sufficient or high level of van Hiele geometry understanding tend to establish a high level of constructivist-based teaching practice. In other words, teachers who are expected level or above of van Hiele geometry understanding tend to instruct effectively with respect to constructivist approach. On the other hand, those who have a low level of van Hiele geometry understanding tend to represent a low level of constructivist-based teaching practice.

This research provides information about the relationship between pre-service teachers' content knowledge and the quality of their instructions by investigating the relationship between the PEMTs' van Hiele geometric thinking level and constructivist-based teaching practice. Pre-service teachers were ideal candidates since they provided information about sufficiency of the teacher education programs in Turkey. Likewise, this research can be organized and then conducted for in-service teachers so that in-service and pre-service teachers with respect to their van Hiele geometric thinking levels and the quality of their instructions can be examined comparatively.

The Ethical Committee Approval

Since this research was conducted before 01.01.2020, it does not require an ethical committee approval.

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Uzun Öz

Giriş

Alan yazında yer alan geometri ile ilgili bazı araştırmalar öğrencilerin çoğunun geometri öğrenemediğini ve geometri bilgilerinin yeterli olmadığını göstermektedir (Clements ve Battissa, 1992; Ubuz ve Ustün, 2003; Usiskin, 1982; van Hiele, 1986; van Hiele-Geldof, 1984). Öğrencilerin geometriye yönelik ihtiyaçlarını karşılamak ve bu alanda başarılı olmalarını sağlamak için özellikle öğretmenlerin uygun öğrenme ortamları hazırlamaları beklenmektedir (Gül-Toker, 2008). Bu doğrultuda gerekli geometri bilgisine sahip olan ve öğretme bilgisine sahip öğretmenler yetiştirmek önemlidir. Bu öğretmenler önceki çalışmalarda vurgulanan ve öğrencilerin öğrenme güçlüğüne sahip olan müfredatın zayıf bir şekilde işlenmesi ve geometri öğretiminde karşılaşılan zorluklarla bahşedilememesi gibi problemleri (Clements 1999; Lehrer 2003, Steele, 2013; Strom vd., 2001) çözebilirler. Bunun yanı sıra, öğretmen adaylarının sahip olduğu bilgiler de gelecekte nasıl öğretmenler olacaklarını şekillendirmektedir (Mayberry, 1983). Dolayısıyla, öğretmen adaylarının sahip oldukları geometri alan bilgilerinin düzeyini ve geometri öğretimlerini ne derecede yapılandırmacı yaklaşıma uygun gerçekleştirebildiklerini anlamak onların geometri öğretimiyle ilgili becerilerindeki ve bilgilerindeki eksiklikleri gidermek ve bu anlamda gelişimlerini sağlamak açısından önemlidir. Bu doğrultuda, bu çalışmanın amacı geleceğin öğretmenleri olarak ilköğretim matematik öğretmen adaylarının van Hiele geometrik düşünme düzeylerini belirlemek, onların geometri öğretim kalitelerini değerlendirmek ve geometrik düşünme düzeyleri ile Yenilenen Öğretimi Gözlem Protokolü (RTOP) boyutlarına (ders tasarımı ve uygulama, içeril ve sınıf kültürü) göre gerçekleşen yapılandırmacı temelli öğretim uygulamaları arasındaki ilişkiyi incelemektir. Çalışmada aşağıdaki sorulara cevap vermek amaçlanmıştır:

1. İlköğretim matematik öğretmen adaylarının Van Hiele geometrik düşünme düzeyleri nedir?
2. İlköğretim matematik öğretmen adaylarının Van Hiele geometrik düşünme düzeyleri ile yapılandırmacı temelli öğretim uygulamaları arasındaki ilişki nasıldır?
   a. İlköğretim matematik öğretmen adaylarının Van Hiele geometrik düşünme düzeyleri ile yapılandırmacı temelli öğretim uygulamaları arasındaki ilişki alan bilgisini öğretmenden öğrenciye transfer etme açısından nasıldır?
   b. İlköğretim matematik öğretmen adaylarının Van Hiele geometrik düşünme düzeyleri ile yapılandırmacı temelli öğretim uygulamaları arasındaki ilişki uygun öğrenme ortamı tasarlama ve uygulama açısından nasıldır?
   c. İlköğretim matematik öğretmen adaylarının Van Hiele geometrik düşünme düzeyleri ile yapılandırmacı temelli öğretim uygulamaları arasındaki ilişki sınıf kültürü açısından nasıldır?

Yöntem

Öğretmen adaylarının geometrik düşünme düzeyleri ile yapılandrmacı öğretim uygulamalarını incelemek ve bunlar arasındaki ilişkiyi ortaya koymak için nicel ve nitel yaklaşımlar birlikte kullanılmış ve nicelin baskın olduğu karma yöntem tercih edilmiştir (Sullivan, 2009). Bu yöntem ile öğretmen adaylarının bu konudaki inanışları ile gerçekte sergiledikleri performans arasındaki ilişkilere odaklanmak hedefleniştir. Bir devlet üniversitesinin lisans programında öğrenim gören 108 üçüncü ve dördüncü sınıf ilköğretim matematik öğretmen adayı bu araştırmaya gönüllü olarak katılmıştır. Bunların, % 53,7'si kız ve % 46,3'ü erkek öğrencidir. Ayrıca, % 56,7'si üçüncü sınıfta ve % 44,4'ü dördüncü sınıfta okumaktadır. Bu öğretmen adayları mezun olduklarında beşinci, altıncı, yedinci ve sekizinci sınıfta matematik öğretmeni olacaklardır.

Bu çalışmada iki adet nicel ölçme aracı kullanılmıştır. Bunlardan birincisi Yenilenen Öğretimi Gözlem Protokolü (RTOP)'dür. RTOP öğretmen adaylarının geometri öğretimi ve özel öğretim yöntemleri derslerinde gerçekleştirdikleri yapılandırmacı temelli öğretim uygulamalarını incelemek için kullanılmıştır. Bu gözlem protokolü, Türkiye'deki eğitim felsefesi yapılandırmacı olduğundan ve öğretimin yapılandırmacı yaklaşıma göre ne derece düzenlendiğini değerlendirmeyi sağlayan standartlaştırılmış bir ölçme aracı olduğundan tercih edilmiştir. Bu bağlamda, RTOP öğretmen adayları tarafından gerçekleştirilen geometri öğretimlerinin kalitesini belirlemek için güvenli bir yöntem olarak sınıf gözlemlerini gerçekleştirmek amacıyla kullanılmıştır. RTOP dersin tasarımı ve uygulaması, içerik ve sınıf kültürü olmak üzere üç temel boyutu kapsamaktadır. İçerik boyutu, 5 maddeden oluşan öneri bilgisi ve yine 5 maddeden oluşan işlemsel bilgi olmak üzere iki alt kategoriden oluşmaktadır. Sınıf kültürü boyutu, 5 maddeden oluşan iletişimsel etkileşim ve yine 5 maddeden oluşan öğrencilerle ilişkiler olmak üzere iki alt kategoriyi kapsamaktadır. Bu doğrultuda, RTOP toplamda 25 maddeden oluşan bir ölçme aracıdır (Temiz ve Topcu, 2011). Belirli bir davranış hiç gerçekleşmediğinde "1" puanı verilirken belirli bir davranış gözlemlenen bireyin davranışları hakkında çok açıklayıcı olduğunda "5" puanı verilmektedir. Yapılan puanlar 0-125 puan arasında değişmektedir. Ayrıca bu ölçek, Temiz ve Topçu'ya (2011) ait bir çalışmada Türkçe'ye çevrilmiştir. Temiz ve Topcu (2011) bu ölçeğin güvenilir ve geçerli olduğunu onaylamıştır. Ölçek için üç boyut belirlenmiş ve birinci, ikinci ve üçüncü seviye için Cronbach alfa güvenilirlik ölçütleri sırasıyla 0,90, 0,86 ve 0,91 olarak bulunmuştur.

İkinci ölçme aracı Van Hiele Geometri Testidir. Bu test Ususkin (1982) tarafından ilköğretim matematik öğretmen adaylarının geometri alan bilgilerini belirlemek için geliştirilmiştir. Testin içinde 25 çoktan seçmeli geometri sorusu bulunmaktadır ve 35 dakikada uygulanmaktadır. İlk beş madde düzey 1'e, ikinci beş madde düzey 2'ye, üçüncü beş madde düzey 3'e, dördüncü beş madde düzey 4'e ve son beş madde düzey 5'e yöneliktir. Ayrıca, testin geçerlik ve güvenirliği için Burger ve Shaughnessy (1986), Fuys, Geddes ve Tischler (1988), Moody (1996), Moran (1993) ve Usiskin (1982) pek çok öğrenciyle çalışmalar yapmıştır. Testin Türkçeye uyarlanması Duatepe (2004) tarafından yapılmış ve Cronbach alfa güvenilirlik ölçütü yeterli bulunmuştur.

Çalışmaya katılmayı kabul eden ilköğretim matematik öğretmen adayları yapılandırmacı yaklaşıma uygun geometri mikro öğretimlerini tasarlamış ve bunları ders kapsamında uygulamıştır. Öğrenciler çalışmada bir kez mikro öğretim gerçekleştirmişlerdir. Araştırmacılar RTOP kullanarak mikro öğretimleri gözlemlemiş ve her bir gözlem 35 dakika sürmüştür. Veri toplama süreci yaklaşık 20 hafta sürmüştür. Ayrıca, mikro öğretimler tamamlandıktan sonra öğretmen adaylarına Van Hiele Geometri testi uygulanmış ve testler araştırmacılar tarafından aşağıdaki kriterler göz önünde bulundurularak puanlanmıştır.

- 1-5 arasındaki (düzey 1) maddelerde sağlanan kriterler için 1 puan
- 6-10 arasındaki (düzey 2) maddelerde sağlanan kriterler için 2 puan
- 11-15 arasındaki (düzey 3) maddelerde sağlanan kriterler için 4 puan
- 16-20 arasındaki (düzey 4) maddelerde sağlanan kriterler için 8 puan
- 21-25 arasındaki (düzey 5) maddelerde sağlanan kriterler için 16 puan (Usiskin, 1982, s. 22).

Öğretmen adaylarının geometrik düşünme düzeylerini değerlendirmek için Gutierrez, Jaime ve Fortuny (1991) tarafından geliştirilen 100 puanlık sayısal ölçek kullanılmıştır. Burada (%0 - %15) arasındaki değerlere göre "Düzey kazanılmamış", (%15-%40) arasındaki değerlere göre "Düzey az kazanılmış", (%40 - %60) arasındaki değerlere göre "Düzey orta derecede kazanılmış", (%60 - %85) arasındaki değerlere göre "Düzey yüksek derecede kazanılmış" ve (%85 - %100) arasındaki değerlere göre "Düzey tamamen kazanılmış" demektir (s. 43). Ayrıca, öğretmen adaylarının geometrik düşünme düzeylerini ve yapılandırmacı temelli öğretim uygulamalarını değerlendirmek için tanımlayıcı istatistik analizi kullanılmıştır. Bunun yanı sıra, öğretmen adaylarının geometrik düşünme düzeyleri ile yapılandırmacı temelli öğretim uygulamalarının her bir alt boyutu (dersin tasarlanması ve uygulaması, içerik ve sınıf kültürü) arasındaki ilişkiyi incelemek için Pearson moment kolerasyon katsayısı çarpımı analizi yapılmıştır. Ayrıca, nicel verileri desteklemek amacıyla katılımcılardan gönüllü olarak seçilen 15 katılımcı ile mülakat yapılmıştır. Katılımcıların yapılandırmacı temelli öğretim uygulamaları ile geometrik düşünme düzeyleri arasındaki ilişkiye ilişkin görüşleri sorulmuştur.

Bulgular, Tartışma ve Sonuç

Araştırmanın sonuçları öğretmen adaylarının geometrik düşünme açısında düzey 3'te olduğunu göstermektedir. Bu düzey ve üstü düzeylere öğretmen adayları tarafından sahip olunması beklenmektedir. Ayrıca, öğretmen adaylarının gözlem protokolünden yüksek puanlar aldığı görülmüştür. Bu doğrultuda, öğretmen adaylarının yapılandırmacı yaklaşıma uygun etkili öğretim yapabildikleri söylenebilir. Araştırmanın bulguları, öğretmen adaylarının geometrik düşünme düzeyleri ile yapılandırmacı temelli öğretim uygulamaları arasında ilişki olduğunu ortaya koymaktadır. Yapılandırmacı temelli öğretim uygulamasının toplam boyutu bazında ve ayrı ayrı ders tasarlama ve uygulama, içerik ve sınıf kültürü alt boyutları bazında öğretmen adaylarının geometrik düşünme düzeyleri ile aralarında pozitif yönde anlamlı ilişki bulunmuştur. Bunun yanı sıra, mülakatlardan elde edilen nitel bulgular nicel bulgular ile tutarlıdır. Bu doğrultuda, yeterli ya da yüksek düzeyde geometrik düşünme yaklaşımına sahip olan öğretmen adaylarının daha etkili bir şekilde yapılandırmacı temelli öğretim uygulamaları kullanma eğiliminde olduğu söylenebilir.